Holographic phase diagram of quark-gluon plasma formed in heavy-ion collisions

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based on I. Ya. Aref'eva, A.A. Bagrov, E.O. Pozdeeva **JHEP**05(2012)117

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Accordingly to AdS/CFT correspondence[1] the black hole formation in AdS_5 is dual to formation of real four-dimensional quark-gluon plasma[2]

The trapped surface can be used as an indication of a black hole formation [3]

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[3] G. W. Gibbons, *Class. Quant. Grav.* 16 (1999) 16771687, hep-th/9809167.
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• To estimate the BH production S. B. Giddings and collaborators[4] have developed the powerful technic of trapped surfaces.
• Gubser, Yarom and Pufu [5] have proposed the gravitational shock wave in AdS_5 as a possible holographic dual for the heavy ion and have related to the area of the trapped surface formed by a collision of such waves to the entropy of matter formed after collision of heavy ions.

• Charged case has been analyzed by I.Ya. Aref'eva, A.A. Bagrov, L.V. Joukovskaya[6]

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[5] S.S. Gubser, S.S. Pufu and A. Yarom *Phys.Rev.* D 78:066014(2008); *JHEP* 0911:050(2009)

[6] I.Ya. Aref'eva, A.A. Bagrov, L.V. Joukovskaya *JHEP* 1003:002 (2010)

The Einstein Equation of ultrarelativistic particle

- The initial point of Gubser and collaborators work is the Einstein equation for the non-change ultrarelativistic particle [7]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{6}{L^2}g_{\mu\nu} = 8\pi J_{\mu\nu}$$

with only one non-zero component of stress-energy tensor:

$$J_{--} = E\frac{z^3}{L^3}\delta(x^1)\delta(x^2)\delta(z-z_0)\delta(x^-)$$

[7] S.S. Gubser, S.S. Pufu and A. Yarom *Phys.Rev.* D 78:066014(2008); *JHEP* 0911:050(2009)

Shock Wave Anzatz

- And making shock wave metric anzatz:

$$dS^2 = dS^2_{AdS_5} + \frac{L}{z}\Phi(x^1, x^2, z)\delta(x^-),$$

$$dS^2_{AdS_5} = \frac{L^2}{z^2}\left(-dx^- dx^+ + (dx^1)^2 + (dx^2)^2 + dz^2\right),$$

the non-zero components of Einstein equation was obtained:

$$\left(\nabla^3_{H_3} - \frac{3}{L^2}\right)\Phi = -16\pi G_5 E \frac{z^4}{L^4}\delta(x^1)\delta(x^2)\delta(z - z_0),$$

$$\nabla^3_{H_3} = \frac{z^2}{L^2}\left[\left(\frac{\partial}{\partial x^1}\right)^2 + \left(\frac{\partial}{\partial x^2}\right)^2 + \frac{\partial^2}{\partial z^2}\right] - \frac{z}{L^2}\frac{\partial}{\partial z},$$

$x^+ = t - x^3$, $x^- = t + x^3$ are light-cone coordinates.

Shock Wave Representation of Ions Collision

• Lin and Shuryak have considered a model where dependence on transversal coordinates x^1, x^2 are absent $\Phi(x^1, x^2, z) = \frac{L}{z}\phi(z)$, (plane wave)[8]. The main reason for such consideration is spreading colliding mass on the surface.

• The Einstein equation for the profile of the plane wave has the form

$$\left(\partial_z^2 - \frac{3}{z}\partial_z\right)\phi(z) = J_{uu}^{\mathrm{WP}}, \quad J_{uu}^{\mathrm{WP}} = -16\pi G_5 \frac{E}{L^2}\frac{z_0^3}{L^3}\delta(z - z_0). \qquad (1)$$

[8] S. Lin, E. Shuryak, *Phys.Rew. D*, 83, 045025 (2011)

Trapped surface

• To find a TS that can be formed in the collision of two plane waves, we must find a solution of Einstein equation (1) satisfying two conditions:

$$\psi(z_a) = \psi(z_b) = 0, \tag{2}$$

$$\psi'(z_a)\frac{z_a}{L} = 2, \qquad \psi'(z_b)\frac{z_b}{L} = -2, \tag{3}$$

$\psi(z) = \frac{L}{z}\phi(z)$, z_a and z_b are assumed to be the boundaries of the TS.

• The solution of Einstein equation (1) was written such that condition (2) is automatically satisfied:

$$\psi(z) = \psi_a(z)\Theta(z_0 - z) + \psi_b(z)\Theta(z - z_0), \tag{4}$$

$$\psi_a(z) = -\frac{4G\pi E(z_0^4/z_b^4 - 1)z_b^4 z_a^3(z^3/z_a^3 - z_a/z)}{L^4(z_b^4 - z_a^4)},$$

$$\psi_b(z) = -\frac{4G\pi E(z_0^4/z_a^4 - 1)z_a^4 z_b^3(z^3/z_b^3 - z_b/z)}{L^4(z_b^4 - z_a^4)}.$$

• But strictly speaking, we cannot call the solution of Einstein equation with considering boundary conditions (2) and (3) the TS.

• Because this surface is assumed to be smooth by definition, while the solution (4) obtained in Shuryak work is *nonsmooth*. We therefore call the solution a quasi-TS.

• For the using of the solutions and TS formation conditions we smooth obtained solutions.

Regularization of TS calculations

- The solution can be decomposed to smooth and nonsmoth parts:

$$\psi_a = \Xi^{(\mathrm{sm})} + \Xi^{\mathrm{nonsm}}. \tag{5}$$

The nonsmooth part of it is

$$\Xi^{\mathrm{nonsm}} = \frac{\mathcal{K}}{z}\left(-\frac{z_b}{z_a^3}\Upsilon_1 - \frac{z_a}{z_b^3}\Upsilon_2\right), \quad \mathcal{K} = \frac{4G\pi E}{L^4}\frac{z_a^3 z_b^3}{z_b^4 - z_a^4}, \tag{6}$$

$$\Upsilon_1 = z^4\Theta(z_0 - z) + z_0^4\Theta(z - z_0),\ \Upsilon_2 = z_0^4\Theta(z_0 - z) + z^4\Theta(z - z_0).$$

- To smooth the solution, we must therefore smooth the nonsmooth part (function Ξ) of the solution.
- We can do this by regularizing the Heaviside step function

$$\Theta(z_0 - z) \approx \Gamma_1 = \frac{\arctan\left(R^3(z_0 - z)^3\right)}{\pi} + \frac{1}{2}, \tag{7}$$

$$\Theta(z - z_0) \approx \Gamma_2 = \frac{\arctan\left(R^3(z - z_0)^3\right)}{\pi} + \frac{1}{2} \tag{8}$$

• We define the functions $F_{a,\mathrm{reg}}$, $F_{b,\mathrm{reg}}$ and consider its at the points $z = z_a$, $z = z_b$ correspondingly:

$$F_{a,\mathrm{reg}}\Big|_{z=z_a} = \frac{z_a}{2L}\left(\frac{d\psi_a}{dz}\Gamma_1 + \frac{d\psi_b}{dz}\Gamma_2\right)\Bigg|_{z=z_a} = 1 + \delta_1,$$

$$F_{b,\mathrm{reg}}\Big|_{z=z_b} = \frac{z_b}{2L}\left(\frac{d\psi_a}{dz}\Gamma_1 + \frac{d\psi_b}{dz}\Gamma_2\right)\Bigg|_{z=z_b} = -1 + \delta_2.$$

• We perform numerical calculations at $R = 10^4$ and obtain that obviously $F_{a,reg} \approx 1$ and $F_{b,reg} \approx -1$.

Charged Shock Wave

• In AdS_5 charged wave shape of point source in Poincare coordinates has the form [9]

$$F = \frac{3\pi\bar{M}}{2a}F_1 + \frac{5\pi\bar{Q}^2}{64a^3}F_2, \quad F_1 = \frac{(8q^2+8q+1) - 4(2q+1)\sqrt{q(1+q)}}{\sqrt{q(1+q)}},$$

$$F_2 = \frac{144q^2+16q-1+128q^4+256q^3-64(2q+1)(q(q+1))^{3/2}}{q(1+q)\sqrt{q(1+q)}}, \quad q = \frac{(x_\perp)^2+(z-z_0)^2}{4zz_0}.$$

• We substitute the shock waves metric to the Einstein equation using the designation $a = L$, $\frac{3\pi\bar{M}}{2L} = \frac{2G_5E}{L}$, $\frac{5\pi\bar{Q}^2}{64L^3} = \frac{5}{48}\frac{G_5EQ^2}{ML^3}$.
We suppose that the shock wave shape depends only from z holographic coordinate and integrate Einstein equation on $x_\perp$ transversal coordinates. We spread charge, mass on transverse extension of the colliding objects.

[9] I. Ya. Aref'eva, A. Bagrov and L. Joukovskaya, *JHEP* 1003:002 (2010).

• The Einstein equation for the charged plane membrane has the form

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi(z) = -16\pi G_5 \mu \frac{z_0^3}{L^3}\delta(z - z_0) - 16\pi G_5 T_{x^-x^-}. \qquad (9)$$

• The charged part of x^-x^- surface density stress-energy tensor component corresponding to plane homogenous source has form

$$-16\pi G_5 T_{x^-x^-} = -\frac{40\pi G_5 Q^2 E}{3L^4 M} \times \qquad (10)$$
$$\times \left(\frac{z^4 z_0(-z^2 + 3z_0^2)}{(-z^2 + z_0^2)^3}\Theta(z_0 - z) + \frac{z_0^5(-3z^2 + z_0^2)}{(-z^2 + z_0^2)^3}\Theta(z - z_0) \right).$$

• We consider the cases $z < z_0$, $z > z_0$ separately using equations:

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi_{q z_0 > z} = -\frac{40\pi G_5 Q^2 E}{3L^4 M}\frac{z^4 z_0(-z^2 + 3z_0^2)}{(-z^2 + z_0^2)^3}, \; z_0 > z;$$

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi_{q z > z_0} = -\frac{40\pi G_5 Q^2 E}{3L^4 M}\frac{z_0^5(-3z^2 + Z_0^2)}{(-z^2 + z_0^2)^3}, \; z > z_0.$$

• In [10] solution $\psi(z) = \frac{L}{z}\phi(z)$ to non-charged membrane Einstein equation constructed by such way that conditions $\psi(z_a) = \psi(z_b) = 0$ satisfied automatically

$$\Psi = \begin{cases} \psi_a = C\left(\frac{z^3}{z_a^3} - \frac{z_a}{z}\right), \; C = -\frac{4\pi G_5 \mu}{L^4} \frac{\left(\frac{z_0^4}{z_b^4} - 1\right) z_b}{\frac{z_b^4 - z_a^4}{z_a^3 z_b^3}}, \; z < z_0 \\ \psi_b = D\left(\frac{z^3}{z_b^3} - \frac{z_b}{z}\right), \; D = -\frac{4\pi G_5 \mu}{L^4} \frac{\left(\frac{z_0^4}{z_a^4} - 1\right) z_a}{\frac{z_b^4 - z_a^4}{z_a^3 z_b^3}}, \; z_0 < z \end{cases}$$

[10] S. Lin, E. Shuryak, *Phys.Rew. D*, 83, 045025 (2011).

• We want that the solution to charged membrane Einstein equation satisfied automatically to the property too.

Thus, the solution part corresponding to charge must satisfy to the property $\psi(z_a) = \psi(z_b) = 0$ and can be represented in the form:

$$\begin{cases} \psi_{aq} = -\dfrac{10}{3}\dfrac{\pi\mu G_5 Q^2}{L^5 M} z_0 z^3 \dfrac{-z_a^2 + z^2}{(-z^2 + z_0^2)(-z_a^2 + z_0^2)}, & z < z_0 \\ \\ \psi_{bq} = \dfrac{10}{3}\dfrac{\pi\mu G_5 Q^2}{L^5 M}\dfrac{z_0^5}{z}\dfrac{-z_b^2 + z^2}{(-z^2 + z_0^2)(-z_b^2 + z_0^2)}, & z_0 < z \end{cases} \tag{11}$$

The complete solution is

$$\Psi = \begin{cases} \psi_a + \psi_{qa} \\ \psi_b + \psi_{qb} \end{cases} \tag{12}$$

• The condition $\psi'(Z_a)\frac{Z_a}{L} = 2,\ \psi'(Z_b)\frac{Z_b}{L} = -2$, gives us the following equations:

$$F_a = -\frac{8\pi G_5 E\left(z_0^4 - z_b^4\right) z_a^3}{L^5(z_b^4 - z_a^4)} - \frac{10}{3}\frac{\pi E G_5 Q^2}{L^6 M}\frac{z_0 z_a^5}{(-z_a^2 + z_0^2)^2} = 1,$$

$$F_b = -\frac{8\pi G_5 E\left(z_0^4 - z_a^4\right) z_b^3}{L^5(z_b^4 - z_a^4)} + \frac{10}{3}\frac{\pi E G_5 Q^2}{L^6 M}\frac{z_0^5 z_b}{(-z_b^2 + z_0^2)^2} = -1.$$

These equations have not analytical solutions. We have looked for numerical solutions.

Phase Diagram

• The comparing of the phase diagrams of the energy (temperature) E versus the chemical potential Q corresponding to the pointlike and the spread cases.

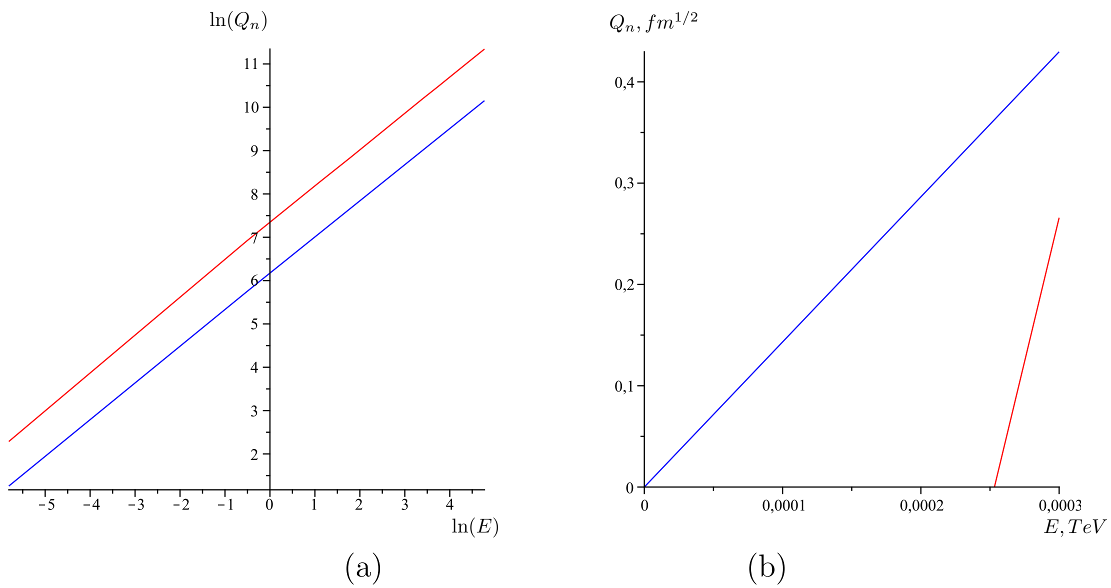

Figure 1: (a) Phase diagram of $\log Q_n$ versus $\log E$ at large E. (b) Phase diagram of E versus Q_n for small E and small Q_n. The blue lines correspond to the pointlike charge, and the red lines correspond to the spread charge.

- The cross point in natural and logarithmic scales

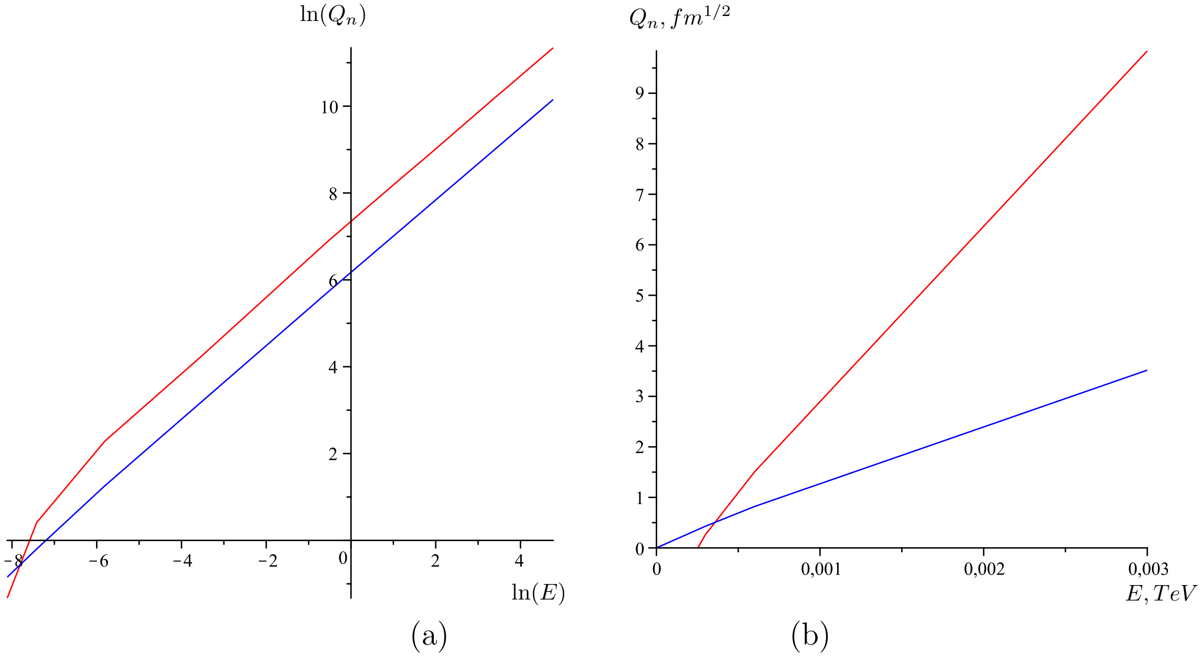

Figure 2: The intersection of the two diagrams in (a) logarithmic and (b) natural scales.

The trapped surface area calculation

Following to Shuryak paper we calculate entropy lower bound, dual to the 'area' of the trapped surface using the formula:

$$S=\frac{2A}{4G_5}=\frac{\int\sqrt{g}dzd^2x_\perp}{2G_5},\quad s\equiv\frac{S}{\int d^2x_\perp}=\frac{L^3}{4G_5}\left(\frac{1}{z_a^2}-\frac{1}{z_b^2}\right). \tag{13}$$

The corresponding graphical representations are in the picture (3):

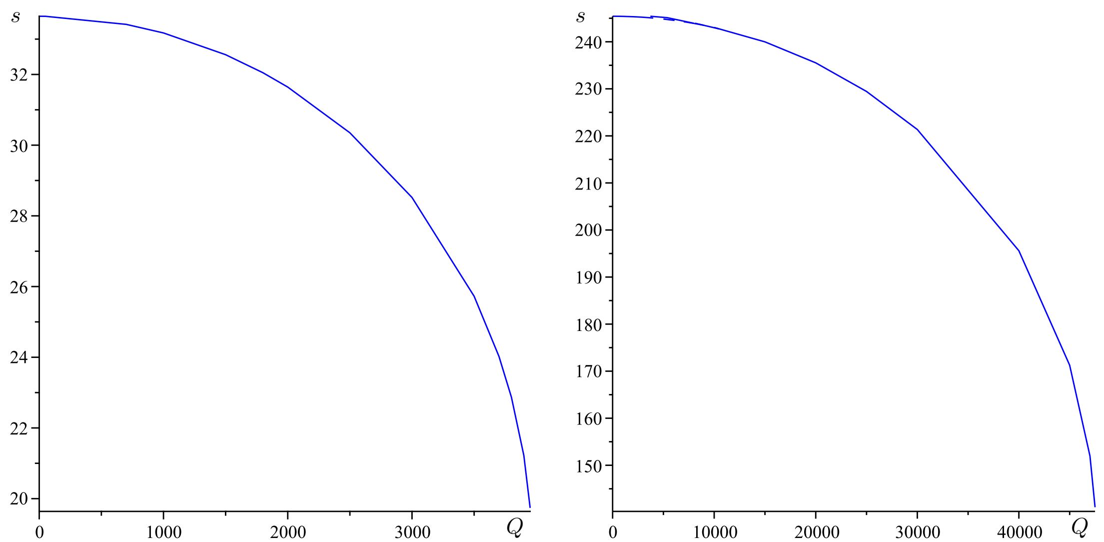

Figure 3: The dynamics of trapped surface area for $E=1\,TeV$ $E=19.7\,TeV$

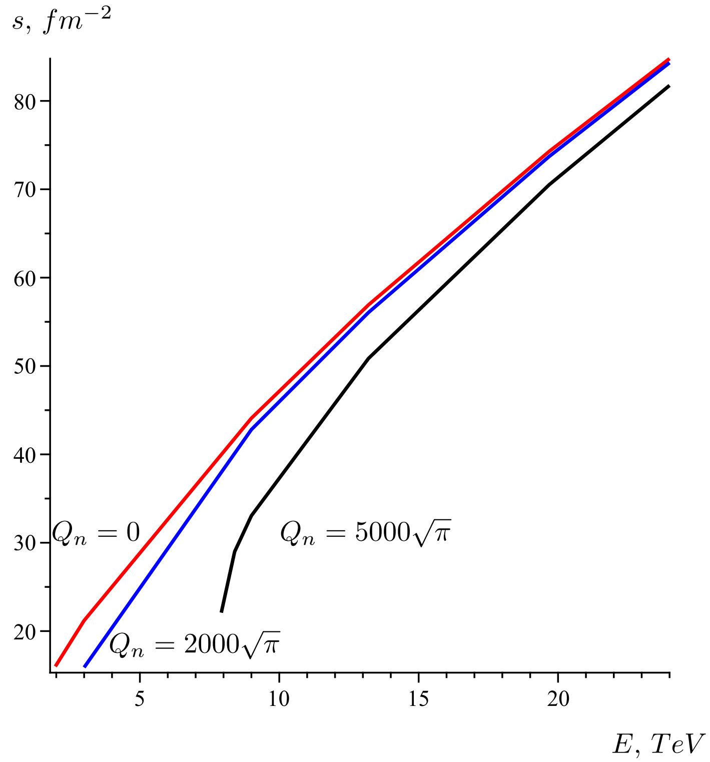

Figure 4: Entropy per square of the colliding objects as a function of energy for different Q: red line, $Q_n = 0\,\mathrm{fm}^{1/2}$; blue line, $Q_n = 2000\sqrt{\pi}\,\mathrm{fm}^{1/2}$; black line, $Q_n = 5000\sqrt{\pi}\,\mathrm{fm}^{1/2}$.

In our case the trapped surface decreases with growth of a chemical potential and increase with growth of energy.

Conclusion

We have considered the heavy ion with chemical potential collision in AdS_5 using Einstein equation and shock waves approach. We have studied the influence of the chemical potential to the trapped surface formation in the charged membranes collision. We obtain the phase diagram of quark-gluon plasma.

Thank you for attention!