Holographic Cut-off flow of Anomalous Transport Coefficients

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Issues

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1. Introduction
   - AdS/CFT and transport coefficients
   - Anomalous transport
2. Flow of the transport coefficients
   - Setup
   - Chiral Magnetic Conductivity
   - Remaining anomalous conductivities: $\sigma_V \,, \sigma_B^{\epsilon} , \sigma_V^{\epsilon}$
   - Gravitational Anomaly
3. Physical interpretation, conclusions and perspectives
   - Interpretation of the results
   - Conclusions

QFT in the hydrodynamic regime

Global equilibrium

$$T_{\mu\nu} = (P+\epsilon)u_\mu u_\nu + P g_{\mu\nu} \quad \text{(Ideal Hydro)} \tag{1}$$

Dual theory: Static Black Hole

Grand canonical ensemble: Global symmetries of the CFT

$$J^\mu = n u^\mu \tag{2}$$

Dual theory: Gauge fields propagating in the bulk

$$\mu \sim A_0(B) - A_0(H) \tag{3}$$

(T, Q_I) of the CFT $\rightarrow$ (T_H, Q_I) of the B-H

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Perturbing the peace

Perturbations with small amplitude

Linear response theory: Kubo formulas → transport coefficients.

Perturbations with small energy

Fluid/gravity correspondence → transport coefficients.

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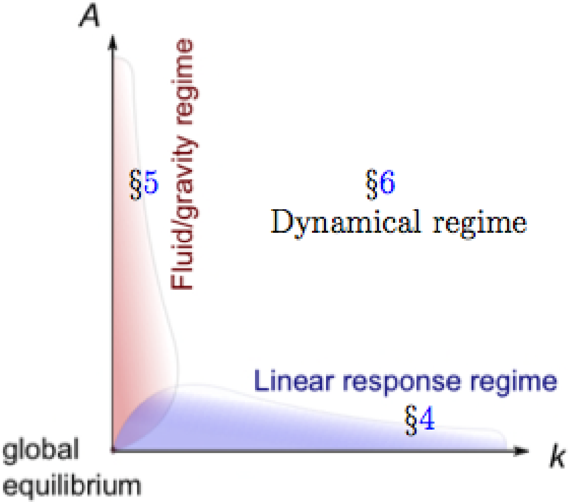
Perturbing the peace II ([Hubeny, Rangamani '10])



Anomalies: Generalities

- Axial anomaly [Bell, Jackiw '69]

$$D_\mu j_5^\mu = \frac{-1}{96\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \tag{4}$$

- Mixed anomaly [Delbourgo, Salam '72]

$$D_\mu j_5^\mu = \frac{-1}{384\pi^2}\epsilon^{\mu\nu\rho\sigma}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\sigma} \tag{5}$$

- Both can be implemented by including Chern-Simons terms in the bulk action, i.e. [Son, Surówka '09]

$$\Delta S \sim \int d^5x\sqrt{-g}\left(\frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}\right) \tag{6}$$

Anomalous transport I

Chiral Magnetic Effect: Appearance of a current in the direction of $\vec{B}$
[Amado, Landsteiner, Pena-Benitez '11]

$$\delta J^\mu = \sigma_B \epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma} \quad (7)$$

$$\delta T^{\mu\nu} = \sigma_B^\epsilon (u^\mu B^\nu + (\mu \to \nu)) \quad (8)$$

$$\sigma_B = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \left\langle J^a J^b \right\rangle (\omega = 0, \vec{p}) \quad (9)$$

$$\sigma_B^\epsilon = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \left\langle T^{0a} J^b \right\rangle (\omega = 0, \vec{p}) \quad (10)$$

where $a, b, c = x, y, z$.

Anomalous transport II

Chiral Vortical Effect: Appearance of a current due to vortices in the fluid ω^m [Amado, Landsteiner, Pena-Benitez '11]

$$\delta J^\mu = \sigma_V \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma \tag{11}$$

$$\delta T^{\mu\nu} = \sigma_V^\epsilon (u^\mu \omega^\nu + (\mu \to \nu)) \tag{12}$$

$$\sigma_V = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \left\langle J^a T^{0b} \right\rangle (\omega = 0, A_0 = 0) \tag{13}$$

$$\sigma_V^\epsilon = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \left\langle T^{0a} T^{0b} \right\rangle (\omega = 0, \vec{p}) \tag{14}$$

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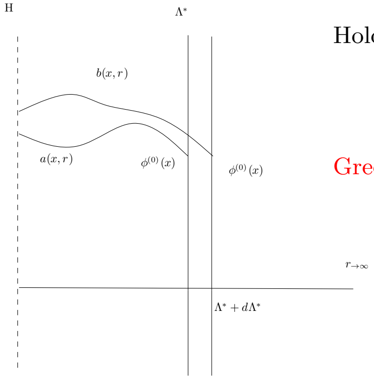
Cutoff flow



Two theories, one equipped with a cutoff Λ^* and the other with $\Lambda^* + d\Lambda^*$ ($S_{\Lambda^*} \neq S_{\Lambda^*+d\Lambda^*}$)

Holographic dictionary: Sourced one-point function

$$< \mathcal{O} >^S_{\Lambda^*} \sim \frac{\delta S^{on-shell}_{B,ren.}}{\delta \phi}(r = \Lambda^*) \tag{15}$$

Green's function $\mathcal{G}_{\mathcal{R}}$: up to first order in the source

$$< \mathcal{O} >^S_{\Lambda^*} = \lim_{r \to \Lambda^*} \mathcal{G}_R(r) \phi(r) \tag{16}$$

Computation...

For the theory living at $\Lambda^* + d\Lambda^*$

$$\mathcal{G}_R(\Lambda^* + d\Lambda^*) \approx \mathcal{G}_R(\Lambda^*) + \frac{1}{\phi(\Lambda^*)} d\Lambda^* \frac{d}{d\Lambda^*} \left(< \mathcal{O} >^S_{\Lambda^*} \right) \tag{17}$$

$$\frac{d\mathcal{G}_R(\Lambda^*)}{d\Lambda^*} = \frac{1}{\phi(\Lambda^*)} \frac{d}{d\Lambda^*} \left(< \mathcal{O} >^S_{r=\Lambda^*} \right) \tag{18}$$

It turns out that $\frac{d}{d\Lambda^*} \left(< \mathcal{O} >^S_{\Lambda^*} \right)$ can be related to the equations of motion of $< \mathcal{O} >^S (r)$ and $\phi(r)$ for the theory defined at Λ^* by formally identifying r with Λ^*!

We can describe the cutoff flow as dynamics in the bulk

The model

The action writes [Son, Surówka '09]

$$\Delta S = \frac{1}{16\pi G}\int_{r<\Lambda^*}\sqrt{-g}\left(-\frac{1}{4}F_{MN}F^{MN} + \frac{\kappa}{3}\epsilon^{MNPQR}A_M F_{NP}F_{QR}\right)$$

Background metric (RN-AdS BB) and A_μ fields

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\bar{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2 \quad (19)$$

$$f(r) = 1 - \frac{ML^2}{r^4} + \frac{Q^2L^2}{r^6}; \qquad A = -\frac{\mu r_H^2}{r^2}dt \quad (20)$$

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Consistent definition of the current [Amado, Landsteiner, Pena-Benitez '11]

$$J^\mu = \frac{\delta S}{\delta A_\mu(r=\Lambda^*)} = \lim_{r\to\Lambda^*}\frac{\sqrt{-g}}{16\pi G}\left(F^{\mu r} + \frac{4\kappa}{3}\epsilon^{r\mu\nu\rho\lambda}A_\nu F_{\rho\lambda}\right) \qquad (21)$$

Anomalous Ward Identity! → $D_\mu J^\mu = \lim_{r\to\Lambda^*}\frac{-\sqrt{-g}\kappa}{48\pi G}\epsilon^{r\mu\nu\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$ (22)

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Flow of the Electric and Chiral Magnetic Conductivities

Perturbations that we switch on: $a_x(x,r); a_z(x,r)$. In momentum space $a_{(x,z)}(k,r) = a(r)e^{-i\omega t + iky}$

Two equations for the current (covariant formula & constitutive formula)

$$j^x \equiv \delta J^x = \frac{\sqrt{-g}}{16\pi G}\delta F^{\mu r} \tag{23}$$

$$\delta J^x = \sigma_E E^x + \sigma_B B^x \equiv \sigma_E \delta F_{0x} + \sigma_B \epsilon(xjk)\frac{\delta F_{jk}}{2} \tag{24}$$

Equations of motion on Σ_r

$$\frac{1}{\sqrt{-g}}\left[-16\pi G \partial_r j^x + \partial_y(\sqrt{-g}\delta F^{yx}) + \partial_t(\sqrt{-g}\delta F^{tx})\right] = -8\kappa \epsilon^{rtxyz} F_{rt}\delta F_{yz} \tag{25}$$

$$\partial_r j^x + \frac{\sqrt{-g}}{16\pi G}\left(ikg^{yy}g^{xx}B^z + i\omega g^{xx}g^{tt}E^x\right) = -\frac{\kappa}{2\pi G}F_{rt}B^x \tag{26}$$

Bianchi identities: $B^z = \frac{k}{\omega}E^x$

Flow of the Electric and Chiral Magnetic Conductivities II

Working with the constitutive equation...

$$\partial_r \delta J^x = \partial_r \sigma_E E^x + \sigma_E \partial_r E^x + \partial_r \sigma_B B^x + \sigma_B \partial_r B^x \tag{27}$$

$$\partial_r B^x = ik\delta F_{rz} = \frac{-ik16\pi G}{\sqrt{-g}} g_{rr} g_{zz} \delta J^z = \frac{-ik16\pi G}{\sqrt{-g}} g_{rr} g_{zz} \left[\sigma_E E^z + \sigma_B B^z\right] \tag{28}$$

...and using the Bianchi identities again...

$$\partial_r B^x = \frac{-ik16\pi G}{\sqrt{-g}} g_{rr} g_{zz} \left[\sigma_E \frac{-\omega}{k} B^x + \sigma_B \frac{k}{\omega} E^x\right] \tag{29}$$

...Finally, one gets a different equation for $\partial_r j^x$

$$\partial_r j^x = E^x \left[\partial_r \sigma_E + \frac{i\omega 16\pi G}{\sqrt{-g}} g_{rr} g_{xx} \left(\sigma_E^2 - \frac{k^2}{\omega^2} \sigma_B^2\right)\right] + \\ + B^x \left[\partial_r \sigma_B + \frac{i\omega 32\pi G}{\sqrt{-g}} g_{rr} g_{xx} \sigma_B \sigma_E\right] \tag{30}$$

Flow of the Electric and Chiral Magnetic Conductivities II

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Computation of the flow for the Electric and Chiral Magnetic Conductivities III

Flow Equations

$$\partial_r \sigma_E = -i\omega \left[\frac{16\pi G}{\sqrt{-g}} g_{rr} g_{xx} \left(\sigma_E^2 - \frac{k^2}{\omega^2} \sigma_B^2 \right) + \frac{\sqrt{-g}}{16\pi G} g^{xx} \left(g^{tt} + \frac{k^2}{\omega^2} g^{yy} \right) \right] \quad (31)$$

$$\partial_r \sigma_B = -i\omega \frac{32\pi G}{\sqrt{-g}} g_{rr} g_{xx} \sigma_B \sigma_E - \frac{\kappa}{2\pi G} F_{rt} \quad (32)$$

$$\partial_r \sigma_E = 0 \quad (33)$$

$$\partial_r \sigma_B = -\frac{\kappa}{2\pi G} F_{rt} = \frac{\mu r_H^2}{2\pi^2 r^3} \quad (34)$$

σ_E, σ_B

$$\sigma_E = Constant, \quad \sigma_B(r) = \frac{\mu}{4\pi^2} \left(1 - \frac{r_H^2}{r^2} \right) \quad (35)$$

- Equation (35) is perfectly consistent with the flow analysis of [Iqbal, Liu '08], implying the universality of σ_E.
- $\sigma_B(r_H) = 0$

Result and emergence of $\mu(\Lambda)$

The energy-momentum tensor reads

$$t^a_b = \lim_{r\to\Lambda^*} \frac{\sqrt{-\gamma}}{8\pi G}(\delta^a_b K - K^a_b) \tag{36}$$

(γ is the determinant of the induced metric and K^a_b is the extrinsic curvature).

Repeating the process...

$$\sigma_V(\Lambda) = \frac{\mu^2}{8\pi^2}\left(1 - \frac{2r_H^2}{\Lambda^2} + \frac{r_H^4}{\Lambda^4}\right) = \frac{1}{8\pi^2}\left[\mu\left(1 - \frac{r_H^2}{\Lambda^2}\right)\right]^2 \tag{37}$$

$$\sigma_B^\epsilon(\Lambda) = \frac{1}{8\pi^2}\left[\mu\left(1 - \frac{r_H^2}{\Lambda^2}\right)\right]^2 \tag{38}$$

$$\sigma_V^\epsilon(\Lambda) = \frac{1}{12\pi^2}\left[\mu\left(1 - \frac{r_H^2}{\Lambda^2}\right)\right]^3 \tag{39}$$

Emerging chemical potential

$$\mu(\Lambda) = \mu\left(1 - \frac{r_H^2}{\Lambda^2}\right) \tag{40}$$

Changing the strategy: transport coefficients as 2-point functions

Gravitational Anomaly

Problem much more complicated: reformulate it as the computation of 2-point functions between the horizon and the cutoff surface, with B.C. at $r = \Lambda$

- Solve the perturbed E.O.Ms in the limit $\omega = 0$ with the bulk-to-boundary propagator normalized at $r = \Lambda$
- Apply the prescription of [Kaminski et al. '09] to compute the retarded correlators (now depending on Λ).
- Use equations (9), (10), (13) and (14) to obtain the desired conductivities.

The Model

$$S = S_{EHM} + S_{\mathcal{A}EM} + S_{\mathcal{A}CS} + S_{\partial} + S_{CSK} \tag{41}$$

$$S_{EHM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda_c - \frac{1}{4}F_{MN}F^{MN}\right] \tag{42}$$

$$S_{\mathcal{A}EM} = \frac{\kappa}{48\pi G}\int d^5x\sqrt{-g}\epsilon^{MNPQR}A_M F_{NP}F_{QR} \tag{43}$$

Gravitational anomaly

$$S_{\mathcal{A}CS} = \frac{\lambda}{16\pi G}\int d^5x\sqrt{-g}\epsilon^{MNPQR}A_M R^A{}_{BNP}R^B{}_{AQR} \tag{44}$$

$$S_\partial = -\frac{1}{8\pi G}\int_\partial \sqrt{-h}K \tag{45}$$

$$S_{CSK} = -\frac{\lambda}{2\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}n_M\epsilon^{MNPQR}A_N K_{PL}D_Q K^L_R \tag{46}$$

- S_{CSK} ensures that the anomalous Ward identity for gauge transformations depends only on the intrinsic curvature tensor on the boundary at $r = \Lambda$.

$$16\pi G J^A = n_B\left[F^{AB} - 8\epsilon^{BACDE}\lambda K_{CF}D_D K^E_F\right]_{r=\Lambda} \tag{47}$$

$$D_\mu J^\mu = -\frac{1}{16\pi G}\epsilon^{opqr}\left[\frac{\kappa}{3}F_{op}F_{qr} + \lambda R^a_{(4)bop}R^b_{(4)aqr}\right]_{r=\Lambda} \tag{48}$$

Issues

- The term $A \wedge R \wedge R$ has not a well defined Dirichlet problem → not trivial to define generic 1-point functions

$$\delta(S_{\mathcal{A}CS} + S_{CS\mathcal{K}}) = -\frac{\lambda}{2\pi G}\int_\partial \sqrt{-h}\epsilon^{mlqr} D_r A_m \delta K_q^v K_{lv} \tag{49}$$

There is not an easy way to avoid the appearance of such boundary term

- The equations for the perturbations, for the sector we are interested in, and in the limit $\omega = 0$, happen to be of second order in r-derivatives!

The approach: Solve the second order equations with infalling B.C. at the horizon and Dirichlet at the boundary. Then take into account the contribution of (49) to the boundary variation of the action.

Physical consistency check: (49) can only affect the correlators $\langle T_t^x J^z\rangle$, $\langle J^x T_t^z\rangle$. The way we treat (49) holographically must make $\langle T_t^x J^z\rangle$, $\langle J^x T_t^z\rangle$ compatible with $\langle T_t^x T_t^z\rangle$ and $\langle J^x J^z\rangle$.

Issues II

- We compute $\langle T_t^x T_t^z \rangle$ and $\langle J^x J^z \rangle$ (not affected by (49))

$$\langle J^x J^z \rangle = 0 \tag{50}$$

$$\langle T_t^x T_t^z \rangle = -ik \frac{\mu(1-u_c)T^2}{12} \tag{51}$$

- These results hint again at an effective $\mu(\Lambda)$ and a non-flowing "effective" temperature.

Consistency Check

Our prescription must lead to correlators $\langle T_t^x J^z \rangle$, $\langle J^x T_t^z \rangle$ consistent with (50), (51).

- In terms of the boundary sources, the contribution of (49) reads ($u = r_H^2/r^2$)

$$-\frac{ik\lambda r_H^2 \epsilon_{\alpha\beta}}{2\pi G L^4} \int_\partial u \frac{f'^2(u)}{f(u_c)} a_\beta^{(0)}(k) \tilde{H}_\beta^{(0)}(-k) \tag{52}$$

Final results

2-Point functions

$$\langle J^x J^z \rangle = \frac{ik\kappa\,\mu\,(1-u_c)}{2G\pi} = -ik\frac{\mu(\Lambda)}{4\pi^2} \tag{53}$$

$$\langle J^x T^z_t \rangle = ik\left(\frac{\mu^2(1-u_c)^2}{8\pi^2} + \frac{T^2}{24}\right) \tag{54}$$

$$\langle T^x_t J^z \rangle = ik\left(\frac{\mu^2(1-u_c)^2}{8\pi^2} + \frac{T^2}{24}\right) \tag{55}$$

$$\langle T^x_t T^z_t \rangle = -ik\left(\frac{\mu^3(1-u_c)^3}{12\pi^2} + \frac{\mu(\Lambda)T^2}{12}\right) \tag{56}$$

- Equations (53)-(56) are compatible with the asymptotic value computed in [Amado, Landsteiner, Pena-Benitez '11] & [Landsteiner, Megias, Melgar, Pena-Benitez '11].

- The flow of the different correlators is consistent with respect to each other $\rightarrow$ Physical Consistency check fulfilled!

Interpretation

Effective $\mu(\Lambda)$

The chemical potential represents the necessary energy to introduce a unit of charge into the system. It depends on the cutoff scale: The unit of charge must have a wavelength of, at most, $\Delta\lambda \sim \frac{1}{\Lambda}$. Therefore, it demands less energy to enter the system and spread into it (thermalize).

Effective T^2

The temperature part remains constant as we move the boundary (Hawking temperature).

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$\dot{\sigma}_E = 0$

The electric conductivity is purely dominated by IR contributions. This fact hints at the reason why it is so difficult to compute these conductivities in non-abelian gauge theories such as QCD. One could expect that $\sigma_E = constant$ is an $N_c \to \infty$ effect, and that $\frac{1}{N_c}$ contributions would make $\dot{\sigma}_E(r)$ become a varying function that approaches 0 as $r - r_H$ increases.

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Conclusions and perspectives

- We have studied the cutoff flow of different transport coefficients, showing that, surprisingly, the anomalous ones depend explicitly on the cutoff.
- The flow can be interpreted as an effective flow of the chemical potential. The flow of the different correlators is consistent with respect to each other and the hypothesis of an effective chemical potential $\mu(\Lambda) = \mu\left(1 - \frac{r_H^2}{\Lambda^2}\right) \equiv \mu(1 - u_c)$ is reinforced, in a non-trivial way, by the results extracted from the terms proportional to λ. The temperature part does not flow.
- The Gravitational anomaly presents difficulties for the Dirichlet problem is not well defined in this case. We have treated the issue in such a way that we have been able to obtain consistent results.