Viscosity Correlators in Improved Holographic QCD

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based on

K. Kajantie, M.K., M. Vepsäläinen, A. Vuorinen, arXiv:1104.5352[hep-ph].

K. Kajantie, M.K., A. Vuorinen, *to be published.*

Motivation

- Heavy-ion collision data from RHIC experiment suggests that strongly coupled quark-gluon plasma should behave as an almost ideal liquid ($\eta/s < 0.2$).
- Perturbative QCD (weakly coupled): $\frac{\eta}{s} \approx 1$, i.e. contradicts to this.
- Lattice calculations: small value, but error bars are huge.
- AdS/CFT correspondence: for two-derivative model $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$, i.e. this suggests that holography could be good tool to understand strongly coupled QGP.
- However, QCD is not the conformal theory. Classical conformal invariance is broken by the quantum fluctuations.
- If we want to understand strongly coupled QGP from holography viewpoint we need to have non-conformal model.

Improved Holographic QCD

- IHQCD (or Kiritsis's model) is the non-conformal bottom-up model.
- start with the background black-hole metric

$$ds^2 = b^2(z)\left(-f(z)dt^2 + d\mathbf{x}^2 + dz^2 f^{-1}(z)\right].$$

- and the action for the dilaton gravity

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{4}{3}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right], \quad \lambda(z) = e^{\phi(z)}.$$

- we break conformal invariance by introducing potential $V(\phi)$ for the dilaton field.

To model QCD choose potential $V(\lambda)$ such that

- **For small λ:** potential must have an expansion

$$V(\lambda) \sim \frac{12}{\mathcal{L}^2}(1 + v_0\lambda + v_1\lambda^2) \qquad \lambda \to 0,$$

  where $\mathcal{L}$ is the AdS radius and the coefficients v_0 and v_1 are determined by matching with the beta function of $N \to \infty$ YM theory.

- **For large λ:** leading term must be

$$V(\lambda) \sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} \qquad \lambda \to \infty,$$

  to have an asymptotically-linear glueball spectrum $m_n^2 \sim n$ and theory to be confining.

Shear viscosity correlator

To compare various models (for example IHQCD and AdS/CFT) we compare real-time shear viscosity correlators

$$G_r(\omega, k=0) = -i \int d^4x e^{i\omega t}\theta(t) \langle [T_{12}(t,\vec{x}), T_{12}(0,0)] \rangle \, ,$$

- Spectral density of the shear correlator

$$\rho_s(\omega) = \mathrm{Im}\, G_r(\omega, k=0)$$

- By Kubo formula shear viscosity η is then

$$\eta = \lim_{\omega \to 0} \frac{\rho_s(\omega)}{\omega}$$

To calculate **spectral density** of the shear viscosity correlator

- introduce perturbations to the background metric

$$g_{12} = \epsilon h_{12},$$

- expand resulting Einstein equations up to 1st order in ϵ

$$\ddot{h}_{12} + \frac{d}{dz}\log(b^3 f)\dot{h}_{12} + \frac{\omega^2}{f^2}h_{12} = 0,$$

- Evaluate full action on the AdS boundary

$$\rho_s(\omega) = \frac{1}{4\pi}s(T)\frac{\omega}{|h_{12}\,(z \to 0)\,|^2},$$

where $s(T)$ is the entropy.

Bulk viscosity correlator

- Consequence of the broken conformal invariance in the IHQCD: non-zero bulk viscosity (so far unmeasured at RHIC, but we know that it exists in the real-world QCD).
- To obtain spectral density of the bulk correlator $\langle T_{ii}, T_{jj}\rangle$ introduce metric perturbations

$$g_{ii} \quad = \quad b^2\left(1+\epsilon h_{ii}\right)$$

- Einstein equations for h_{ii}

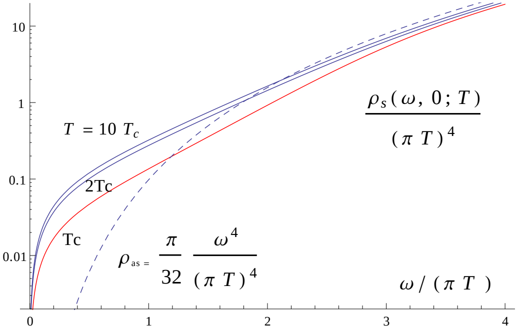
$$\ddot{h}_{ii}+\frac{d}{dz}\log(b^3 f X^2)\dot{h}_{ii}+\left(\frac{\omega^2}{f^2}-\frac{\dot{f}\,\dot{X}}{fX}\right)h_{ii}=0, \qquad X=\frac{\beta}{3\lambda},$$

- Spectral density of the bulk viscosity is then

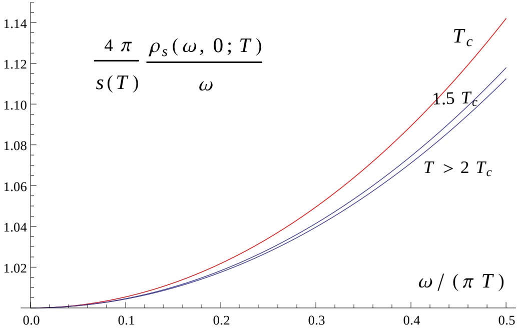
$$\rho(\omega)=\frac{1}{4\pi}s(T)6X^2(z_h)\frac{\omega}{|h_\omega(z\to 0)|^2}.$$

Shear Spectral Density

The spectral function $\rho_s(\omega)/(\pi T)^4$ in units of $\mathcal{L}^3/(4\pi G_5)$

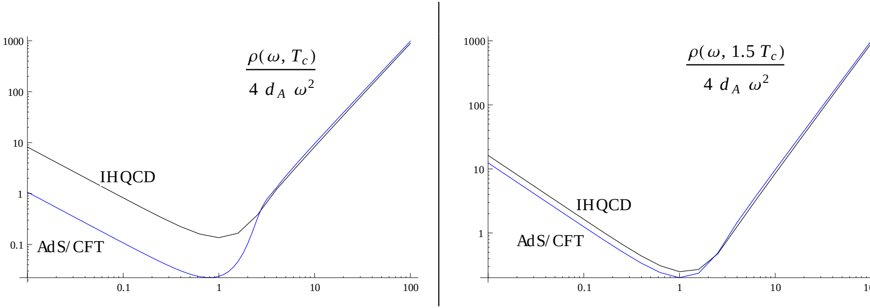


How is this connected with the $\frac{1}{4\pi}$ prediction?



i.e. it is of course satisfied (IHQCD is a two derivative model).

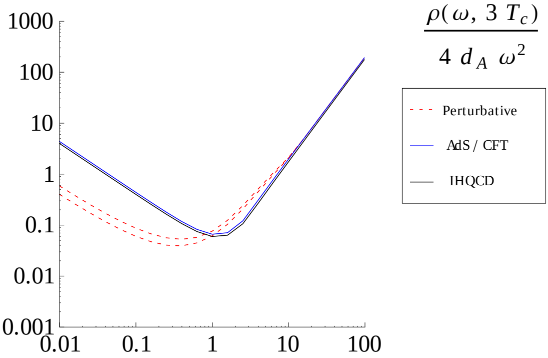
For temperatures close to the T_c and small ω
we can observe the difference between the IHQCD and the conformal theory



- Difference disappear for temeperatures $> 1.5\,Tc$.
- Question: how to compare these results with the QCD?

Comparison with the perturbative QCD

Perturbative QCD is the important tool in QCD, however it works only in weakly coupled region, at high energies. For example at $T = 3Tc$

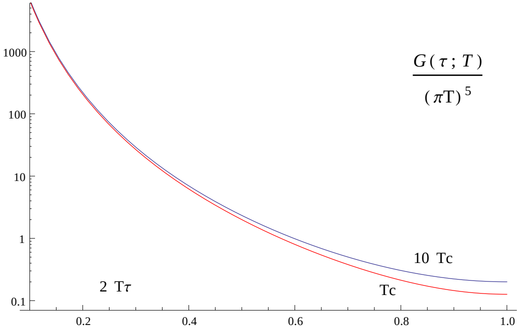


i.e. we can compare our IHQCD results with the pQCD calculations only in the region where IHQCD results approach those of the AdS/CFT.

Euclidean correlator

Imaginary time correlator (i.e. what we observe in lattice QCD)

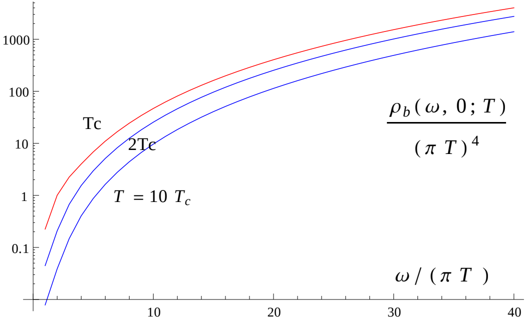
$$G(\tau, T) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega; T) \frac{\cosh\left[(1 - 2T\tau)\frac{\pi}{2}\omega\right]}{\sinh\left(\frac{\pi}{2}\omega\right)}$$



- We can see the difference from the AdS/CFT prediction in the region of the low temperatures.
- on-going work to make the comparison with the lattice data.

Bulk Spectral Density

IHQCD spectral density of the bulk viscosity correlator



Problems:

- slow convergence- numerically challenging in large-ω region
- hard to compare with lattice data - in lattice usually $\langle T_{\mu\mu}, T_{\nu\nu}\rangle$ is calculated

Conclusions

- We have used nonconformal holographic model of QCD to calculate correlators in the shear and the bulk channel.
- In the shear channel we observe strong effect of non-conformality in the low temperature region.
- Bulk channel is completely new, thus we expect our results to be of interest.
- We tried to compare our results with the pQCD and we work on the comparison with lattice.
- Not so easy as it may look:
  - Perturbative QCD holds in the region where the IHQCD coincides with the AdS/CFT results.
  - Lattice data have big error bars (and thus hard to say whether IHQCD is an improvment) or we calculate different correlator.