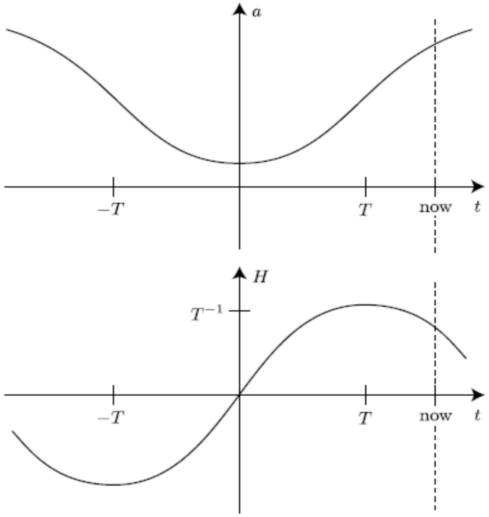
The notorious infinities

- In cosmology:
  - Inflationary spacetimes are not past-complete
  - Avoid Big Bang? *The null energy condition needs to be broken*



- In gravity:
  - For quantization, we need an UV completion of GR
  - Modify GR? *But we don't want ghosts*

- The way towards resolution is nonlocal
  - Only nonlocal gravity may be both singularity and ghost free
  - That is where String theory, LQG, CDT, H-L all hint at…

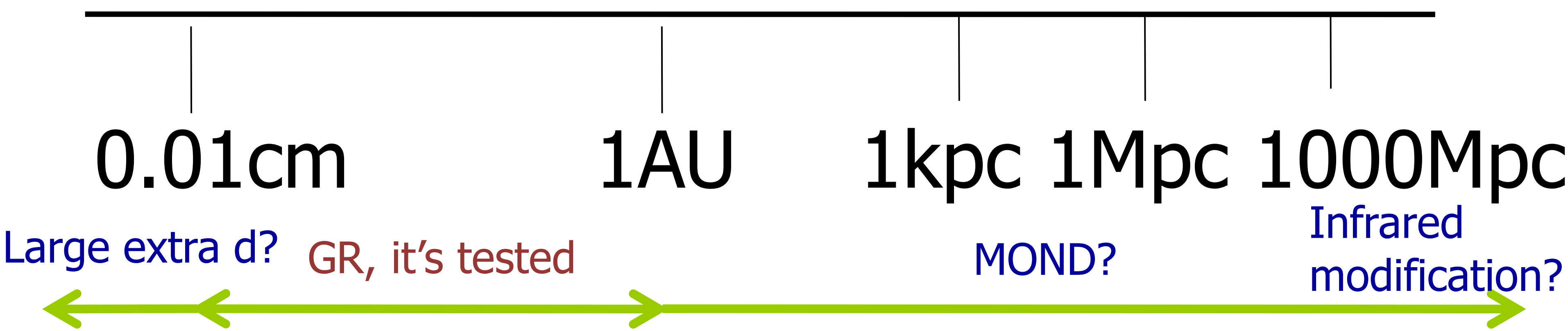
Why modify gravity?

Problems with GR:

- **Theoretical**: - **quantization**
- **singularities**

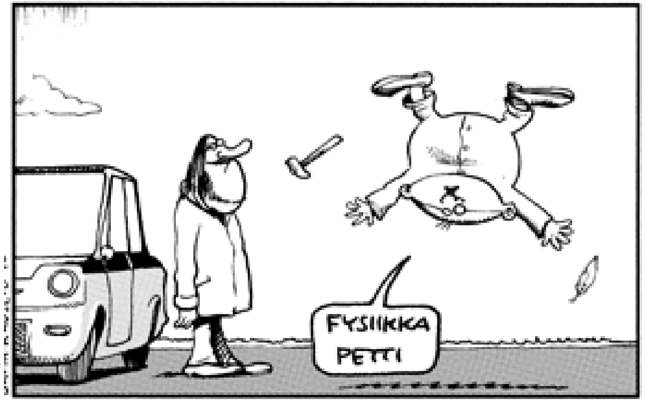
- **Observational**: - **inflation**
  - **dark matter**
  - **dark energy**

Freedom for modifications:



How modify gravity?

$$S_{EH} = \frac{1}{16\pi\, G}\int d^4x\sqrt{-g}R + S_M(g^{\mu\nu},\psi)$$



- Straightforward extension of the Einstein-Hilbert action:

$$S = \int d^4x\sqrt{-g}F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$$

- Results in a higher derivative theory

- Generically, they are unstable **[Ostrogradski (1850)]**

- Energy unbounded below: a ghost **[Woodard, Lect. Notes Phys.720 (2007)]**

Two ways to avoid this:

- **Introduce NO extra derivatives *or***
- **Introduce INFINITELY MANY of them**

<u>The most general</u> quadratic theory

$$
\begin{aligned}
S_q \;=\; & \int d^4x\sqrt{-g}[RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R_\mu^\nu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
& + R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
& + R_\lambda^\rho F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
& + R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
& + R_\mu^{\nu_1\rho_1\sigma_1}F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}]
\end{aligned}
$$

- In flat space, using symmetries, the Bianchi identities, and a lot of work the full propagator boils down to

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

$$a(\Box) = 1 - \frac{1}{2}F_2(\Box)\Box - 2F_3(\Box)\Box$$

$$c(\Box) = 1 + 2F_1(\Box)\Box + \frac{1}{2}F_2(\Box)\Box$$

Gravitons: Poles of P2
Scalar particles: Poles of Ps
- There should be just of one each: extra poles are inevitably ghosts!
  (partial fraction decomposition theorem)

[Biswas, Gerwick, TK, Mazumdar PRL (2012)]

The propagator

- Inverting the field equations for the quadratic action we obtained

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}\,.$$

$$\Pi^{-1\,\lambda\sigma}_{\mu\nu} h_{\lambda\sigma} = \kappa\tau_{\mu\nu}$$

- P2 is the spin-2 projector
- P0 is a scalar projector
- One spin-1 and one spin-0 mode is absent

- General relativity: a=c=1. There occurs a subtle cancelling.

- f(R) gravity: a=1, c=1+mk^2. An extra scalar pole, can be healthy.

- Ricci-squared gravity: a=1+m*k^2. An extra spin-2 pole – the Weyl ghost!

- Theories with c(☐)=a:
  - If a(0)=1, GR recovered in the IR
  - If a(☐) is an entire function, no additional poles
  - If a(☐) grows suitably with k, we improve the UV

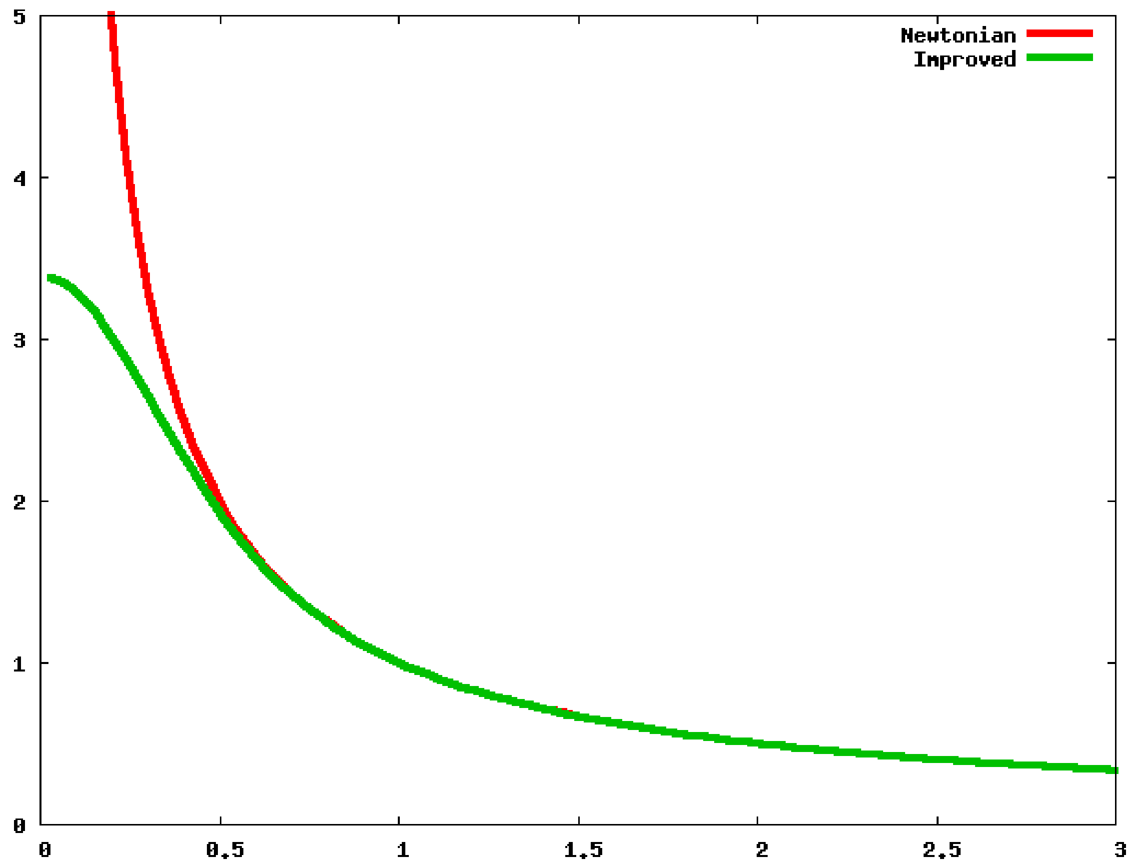
$$\Pi^{\mu\nu}{}_{\lambda\sigma} = \frac{1}{k^2 a(-k^2)}\left(P^2 - \frac{1}{2}P_s^0\right)$$

The Newtonian limit

- In the a=c case, the solution at the Newtonian limit is

$$\Phi(r) \square \frac{m}{M_p^2}\int d^3p\,\frac{e^{i\vec{p}\vec{r}}}{p^2 a(-p^2)} = \frac{4\pi m}{r M_p^2}\int \frac{dp}{p}\,\frac{\sin pr}{a(-p^2)}.$$

- Let us look at the example $a(\square) = e^{-\square/M^2}$



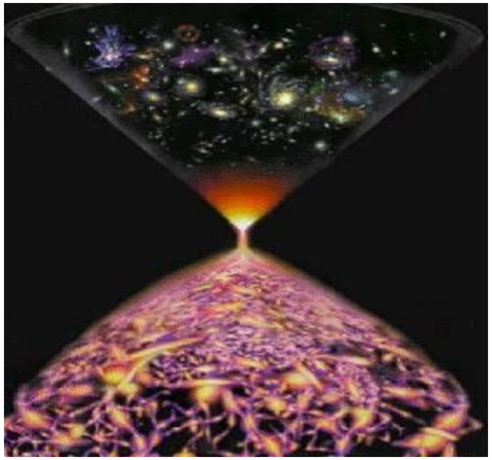
An example: nonsingular cosmology

- Consider the toy model:

$$S = \frac{M_p^2}{2}\int d^4x\,\sqrt{-g}F(R) \qquad F(R) = R + \sum_{n=0}^{\infty}\frac{c_n}{M_*^{2(n+1)}}R\Box^n R\,.$$

If the series doesn't truncate, one may
- exorcise ghosts from the spectrum
- bestow asymptotic freedom

[Biswas, Mazumdar, Siegel: JHEP (2006)]



Since gravitational force then disappears at high curvature
- the Big Bang singularity may be resolved
- It is replaced by a bounce

[Biswas, Koivisto, Mazumdar: JCAP (2010)]

$$\Box R = \lambda R + \mu$$

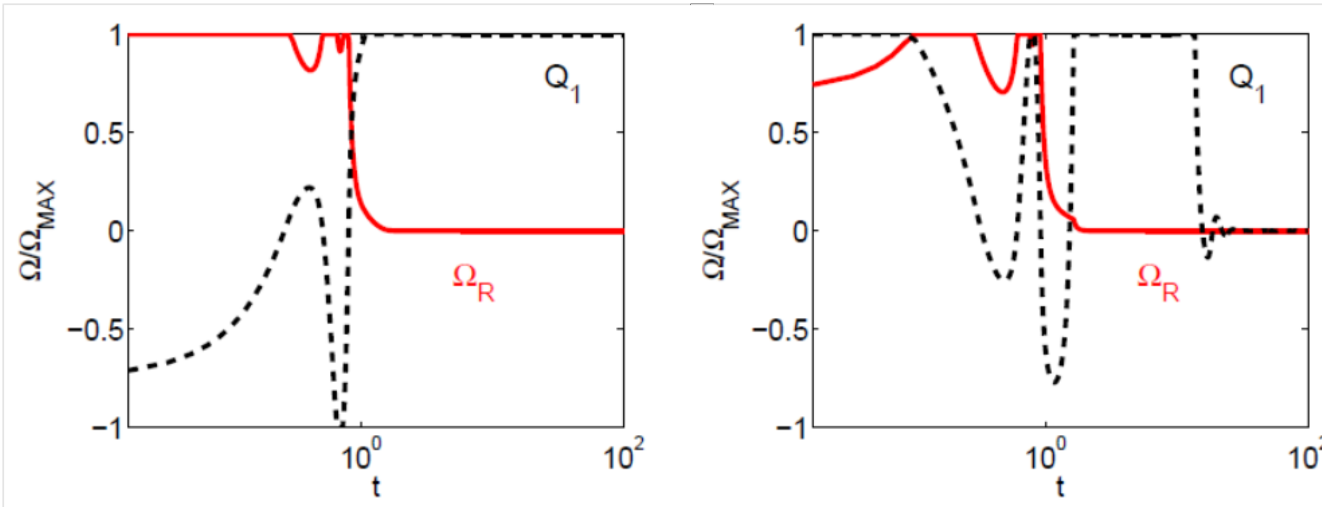
A simple ansatz allows exact solutions!
- We found analytic (a(t)=cosh(t)), series form and numerical solutions
- Applicable also for perturbations

[Biswas, Kosheles, Mazumdar, Vernov:: JCAP (2012)]

Dynamics in nonsingular cosmology

- A dynamical system analysis reveals that we need
  λ>0 to avoid a super-inflating (non-GR) attractor
  μ<0 , thus Λ>0 to end in a de Sitter (GR) attractor

Λ<0



Λ>0

Background doesn't see the details of the theory:

$$\rho \;=\; 9\frac{\lambda - 2c_0\mu/M_*^2}{3\lambda^2-\mu}\left[H\left(H\lambda+2\ddot{H}\right)+6H^2\dot{H}-\dot{H}^2\right]+\frac{1-6c_0\lambda/M_*^2}{12\lambda^3-4\lambda\mu}\mu^2$$

Superhorizon perturbations evolve smoothly and freeze:

$$\delta^{(4)}+7H\delta^{(3)}+(8\dot{H}+12H^2+\lambda)\ddot{\delta}+\left(7\ddot{H}+4H(6\dot{H}+\lambda)\right)\dot{\delta}=0$$

- Explicit bouncing mechanism can be embedded in a cyclic scenario
- This can reconcile AdS space with our universe!

-Blue solid line: the scale factor

-Red solid line: kinetic energy

-Dashed line: potential energy

Conclusions

- **The most general metric theory**
  - Finite-derivative models: either ghosts (Weyl) or non-renomalizable (f(R))
  - Infinite-derivative models: may avoid both singularities and ghosts
- **Nonlocal gravity**
  - May recover GR in the IR and asymptotic freedom in the UV
  - Provide geodesic completion of cosmology by bouncing

- **References:**

Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity.
By Tirthabir Biswas, Tomi Koivisto, Anupam Mazumdar.
JCAP 1011 (2010) 008.

Towards singularity and ghost free theories of gravity.
By Tirthabir Biswas, Erik Gerwick, Tomi Koivisto, Anupam Mazumdar.
Phys.Rev.Lett. 108 (2012) 031101.