Four-dimensional Black Holes in N=2 Supergravity

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Thanks to the collaboration with:

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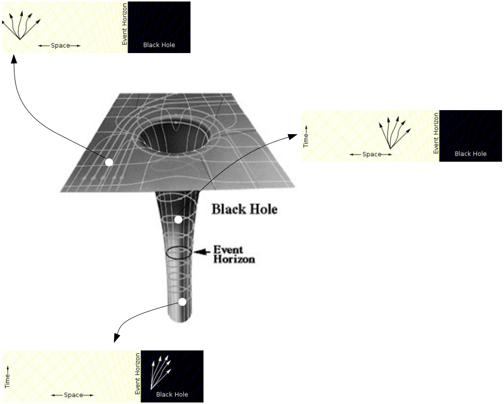
Black hole basics

Black holes

- The main feature of a BH is a region where the spacetime curvature becomes infinite. This is a gravitational singularity and must be hidden within event horizons (*cosmic censorship*)



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- Black holes are completely characterized by their charge (Q), angular momentum (J) and mass (M). Their externally observable features are not affected by the constituent matter, the birth process and objects at infinity (*no hair theorem*)

- The thermodynamical properties of a black hole are related to the spacetime geometry [BEKENSTEIN, HAWKING]:

Entropy	$S = \frac{A_h}{4}$
Temperature	$T = \frac{\kappa}{2}$

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Charged black holes

They appear in Einstein-Maxwell theories where gravity is coupled to e.m. fields:

$$\mathcal{L} = R - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}$$

Charges: $\quad p \propto \int_{S^2} \mathcal{F} \qquad q \propto \int_{S^2} \star\mathcal{F}$

For static spherically symmetric asymptotically flat solutions the ansatz is:

$$\begin{aligned} ds^2 &= e^{2U}dt^2 - e^{-2U}\gamma_{mn}dx^m dx^n \\ \gamma_{mn}dx^m dx^n &= \frac{c^4}{\sinh^4 c\tau}d\tau^2 + \frac{c^2}{\sinh^2 c\tau}d\Omega^2_{(2)} \end{aligned}$$

Charged black holes

$$\mathrm{d}s^2 = \mathrm{e}^{2U(\tau)}\mathrm{d}t^2 - \mathrm{e}^{-2U(\tau)}(c^4 \sinh^{-4}(c\tau)\mathrm{d}\tau^2 + c^2 \sinh^{-2}(c\tau)\mathrm{d}\Omega^2)$$

c is extremality parameter and it holds [GIBBONS,KALLOSH,KOL]: $c^2 = 2ST$.
The relation $\tau \leftrightarrow r$ is: $\sinh^{-2}(c\tau) = (r - r^-)(r - r^+)$

$$r^\pm = r_\mathrm{h} \pm c$$

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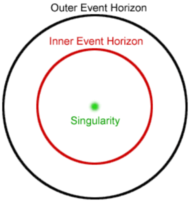
$$r^{\pm} = r_{\mathrm{h}} \pm c$$

- General (non-extremal) Reisnerr-Nordström solution:

$$c = \sqrt{M^2 - (q^2 + p^2)}\,, \qquad e^{2U} = \frac{(r - r^-)(r - r^+)}{r^2}$$



outer: $r^+ \leftrightarrow \tau \to -\infty$

inner: $r^- \leftrightarrow \tau \to +\infty$

Charged black holes

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- Static extremal BHs: $c = 0$, $e^U = 1 - \frac{\sqrt{q^2 + p^2}}{r}$, $\tau \sim -\frac{1}{r}$
  - $M^2 = q^2 + p^2$
  - Finite non-vanishing entropy but zero temperature
  - $S^2 \otimes \mathrm{AdS}_2$ near horizon geometry

An extremal static BH is completely defined by $\mathbf{Q} = (\mathbf{q}, \mathbf{p})$

BHs in Einstein-Maxwell-scalars theories

$$\mathcal{L} = -R(G) + 2g_{ab}(\phi)\partial_\mu \phi^a \partial^\mu \phi^b + f_{IJ}(\phi)\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^{J\,\mu\nu} + \frac{1}{2}k_{IJ}(\phi)\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^J{}_{\rho\sigma}$$

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1. Assume spherical symmetry: all the fields depend on $r \leftrightarrow \tau$
2. Consider static, BH solutions for the metric $G_{\mu\nu}$

   $$\mathrm{d}s^2 = \mathrm{e}^{2U(\tau)}\mathrm{d}t^2 - \mathrm{e}^{-2U(\tau)}\gamma_{mn}dx^m dx^n$$

3. Solve eqm for the vector fields
4. Integrate over $\mathbb{R}_t \times S^2$

$$\Rightarrow \ \mathcal{L}_{\text{eff}} = (U'(r))^2 + g_{ab}\phi'(r)^a\phi'(r)^b - \mathrm{e}^{2U}\ V_{\text{bh}}(\phi, Q)\ + c^2$$

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$$-V_{\text{bh}}(\phi, Q) = f^{\mathsf{IJ}}(\phi)\left(q_{\mathsf{I}} - h_{\mathsf{I}K}(\phi)\,p^K\right)\left(q_{\mathsf{J}} - h_{\mathsf{J}K}(\phi)\,p^K\right) + \mathfrak{f}_{\mathsf{IJ}}(\phi)p^{\mathsf{I}}p^{\mathsf{J}}$$

Field equations:

$$\text{eqm}\begin{cases} U'' + \mathrm{e}^{2U} V_{\mathsf{bh}} = 0 \\ (g_{ab}\phi^{b\,\prime})' - \partial_a g_{bc}\phi^{b\,\prime}\phi^{c\,\prime} + \mathrm{e}^{2U}\partial_a V_{\mathsf{bh}} = 0 \end{cases}$$

$$\text{constraint}\left\{\, (U')^2 + g_{ab}\phi^{a\,\prime}\phi^{b\,\prime} + e^{2U} V_{\mathrm{bh}} = c^2 \right.$$

The scalars evolve from $r = \infty$ to the horizon (r_{h}) driven by V_{bh}.

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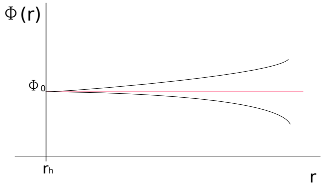
The scalars evolve from $r = \infty$ to the horizon (r_{h}) driven by V_{bh}.

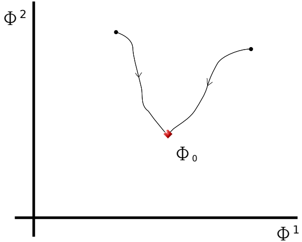
In the extremal case ($c = 0$):

Attractor mechanism [FERRARA,KALLOSH,STROMINGER]

The scalars on the horizon assume a fixed value that extremizes the BH potential $\Rightarrow$ BH geometry depends only on the charges



- The asymptotic values of the scalars turn out to be irrelevant



- $\phi_0 = \phi_0(Q)$ is an attractor in the scalar manifold

Attractor mechanism:

scalar hairs do not affect the BH geometry

$$A_h = A_h(\phi^a(r_h)) = A_h(Q)$$

$$r \to r_h : \phi^a(r) \to \phi_0^a = \phi^a(Q), \quad \phi'(r) \to 0$$

Black holes in N=2, D=4 ungauged supergravity

N=2 Supergravity in 4D

- Multiplet content of the full theory:
  - Supergravity multiplet: $(e^i_\mu,\, \psi^A_\mu,\, \mathcal{A}^0_\mu)$ $\qquad A = 1, 2$
  - n_v Vector multiplets: $(A^a_\mu,\, \lambda^{a\,A},\, z^a)$ $\qquad a = 1, .., n_V$
  - n_H Hypermultiplets: (χ^α, ϕ^u) $\qquad \alpha = 1, .., 2n_H, \;\; u = 1, .., 4n_H$

  Since irrelevant in our discussion, we put to zero the fermion fields

  and omit the hypermultiplets

- The Lagrangian we deal with is (ungauged theory):

$$\mathcal{L} = -R(G) + 2g_{a\bar{b}}(z)\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \mathrm{Im}\,\mathcal{N}_{IJ}(z)\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^{J\,\mu\nu} + \mathrm{Re}\,\mathcal{N}_{IJ}(z)\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^J{}_{\rho\sigma}$$

$\Rightarrow$ **Structure of a Maxwell-Einstein-scalars theory**

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$$I = (0, a) \quad a = 1, \ldots, n_V$$

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- Geometry of the scalar manifold: (very) special

$$F = F(X^I) \qquad z^a = \frac{X^a}{X^0}$$

$$g_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \bar{z}^{\bar{b}}}K \qquad K = -\ln\left[\mathrm{i}(X^I, \partial_{X^I}F)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\overline{\begin{pmatrix}X^I\\ \partial_{X^I}F\end{pmatrix}}\right]$$

$$\mathcal{N}_{IJ} = \overline{\partial_{X^I}\partial_{X^J}F} + 2\mathrm{i}\frac{\mathrm{Im}(\partial_{X^I}\partial_{X^K}F)\,\mathrm{Im}(\partial_{X^J}\partial_{X^M}F)X^M X^K}{\mathrm{Im}(\partial_{X^M}\partial_{X^K}F)X^M X^K}$$

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- By assuming spherical symmetry and staticity, solving for the vectors and integrating $\Rightarrow$ effective 1D Lagrangian:

[FERRARA,GIBBONS,KALLOSH]

$$\mathcal{L}_{\text{eff}} = (\dot{U}(\tau))^2 + g_{a\bar{b}}\dot{z}^a(\tau)\dot{\bar{z}}^{\bar{b}}(\tau) - \mathrm{e}^{2U} V_{\text{bh}}(z, \Gamma) + c^2$$

$$\dot{} = \frac{\mathrm{d}}{\mathrm{d}\tau} \qquad \Gamma = (p^I \;\; q_I) \propto \int_{S^2} \mathcal{F}^I \oplus \frac{\partial \mathcal{L}}{\partial \mathcal{F}^I}$$

$$-V_{\text{bh}} = -\tfrac{1}{2}\Gamma^\Lambda\Gamma^\Sigma\mathcal{M}_{\Lambda\Sigma} = \tfrac{1}{2}(p^I \;\; q_I)\begin{pmatrix} (\mathfrak{I} + \mathfrak{R}\mathfrak{I}^{-1}\mathfrak{R})_{IJ} & -(\mathfrak{R}\mathfrak{I}^{-1})_I{}^J \\ -(\mathfrak{I}^{-1}\mathfrak{R})^I{}_J & (\mathfrak{I}^{-1})^{IJ} \end{pmatrix}\begin{pmatrix} p^I \\ q_I \end{pmatrix}$$

$$= |\mathcal{Z}|^2 + 4g^{a\bar{b}}\partial_{z^a}|\mathcal{Z}|\partial_{\bar{z}^{\bar{b}}}|\mathcal{Z}|$$

$$\mathfrak{R}_{IJ} = \mathrm{Re}\,\mathcal{N}_{IJ}\,, \qquad \mathfrak{I}_{IJ} = \mathrm{Im}\,\mathcal{N}_{IJ}$$

central charge: $\mathcal{Z} = \mathrm{e}^{K/2}(p^I\partial_{X^I}F - q_I X^I)$

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Field equations (second order):

$$\text{eqm}\begin{cases} \ddot{U} + \mathrm{e}^{2U}V_{\mathrm{bh}} = 0 \\ \ddot{z}^a + g^{a\bar{b}}\partial_{z^c}g_{d\bar{b}}\dot{z}^c\dot{z}^d + \mathrm{e}^{2U}g^{a\bar{b}}\partial_{\bar{z}^{\bar{b}}}V_{\mathrm{bh}} = 0 \end{cases}$$

$$\text{constraint}\left\{\, \dot{U}^2 + g_{a\bar{b}}\dot{z}^a\dot{\bar{z}}^{\bar{b}} + e^{2U}V_{\mathrm{bh}} = c^2 \right.$$

First-order formalism

- Based on the rewriting:[CERESOLE,DALL'AGATA & PERZ ET AL.]

$$-\mathrm{e}^{2U}V_{\mathrm{bh}} = (\partial_U Y)^2 + 4g^{a\bar{b}}\partial_{z^a}Y\partial_{\bar{z}^{\bar{b}}}Y - c^2$$

Generalized Superpotential $Y = Y(U, z; \Gamma; c) > 0$

$$\Rightarrow \quad \mathcal{L}_{\mathrm{eff}} = \dot{U}^2 + g_{a\bar{b}}\dot{z}^a\dot{\bar{z}}^{\bar{b}} - \mathrm{e}^{2U}V_{\mathrm{bh}} + c^2$$

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Sum of squares $\quad \left(\dot{U} \pm \partial_U Y\right)^2 + \left|\, \dot{z}^a \pm 2 g^{a\bar{b}} \partial_{\bar{z}^{\bar{b}}} Y \,\right|^2$

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$\Rightarrow$ Extremizing: $\begin{cases} \dot{U} = \pm\partial_U Y \\ \dot{z}^a = \pm 2g^{a\bar{b}}\partial_{\bar{z}^{\bar{b}}} Y \end{cases}$ First-order flow equations

sign depends on conventions

First-order formalism

$$\dot{U} = \pm Y$$

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First-order flow equations

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- If $Y = \mathrm{e}^U |\mathcal{Z}(z;\Gamma)|$, $c = 0$, killing spinor eqs implied $\Rightarrow$

  extremal supersymmetric BH [FERRARA,KALLOSH,STROMINGER]

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- If $Y(U, z; \Gamma; c) \neq \mathrm{e}^U |\mathcal{Z}| \neq \mathrm{e}^U \mathcal{W} \Rightarrow$ non-extremal BH [PERZ,SMYTH,VAN RIET,VERCNOCKE]

Finding the solutions

Solving the system of eqm may turn out not trivial at all

⇒ Depending on the type of BHs different approaches developed:

- Supersymmetric: Stabilization equation [BEHRNDT,LÜST,SABRA; DENEF]
- Non-supersymmetric:
  - Extremal
    - Change the sign of a subset of the charges [TRIPATHY,TRIVEDI; KALLOSH,SIVANANDAM,SOROUSH]
    - Timelike dimensional reduction of black hole solutions obtained as geodesic in the (augmented) scalar manifold [BREITENLOHER,MAISON,GIBBONS]
    - Stabilization equation [PG,GOLDSTEIN,KATMADAS,PERZ]
  - Non-extremal
    - Harmonic deformation of the spacetime metric in 5D and reduction to 4D [MOHAUPT,VAUGHAN]
    - Ansatz for the solutions in 4D by deforming the extremal ones [PG,ORTÍN,PERZ,SHAHBAZI]

Stabilization equation and H-FGK formalism

New unknowns

The main idea is to trade the physical fields for the functions in terms of wich they are expressed:

$$\{z^a(H)\,,\; U(H)\} \qquad \longleftrightarrow \qquad \{H^I(\tau), H_I(\tau)\}$$

and "assume" they are connected by

$$\mathrm{Im}\left[\mathrm{e}^{-U-\mathrm{i}\alpha}\,\mathrm{e}^{-K/2}\begin{pmatrix} X^I \\ \partial_{X^I} F \end{pmatrix}\right] = \begin{pmatrix} H^I(\tau) \\ H_I(\tau) \end{pmatrix}$$

Stabilization equation

One solves for $\hat{\Omega} = \hat{\Omega}(H) \equiv e^{-U} e^{-\mathrm{i}\alpha}\Omega \quad \Rightarrow$

$$z^a = \frac{\hat{\Omega}^a}{\hat{\Omega}^0}$$

$$\mathrm{e}^{-2U} = \tfrac{1}{2}\langle \hat{\Omega}, \bar{\hat{\Omega}} \rangle$$

New equations

Now the unknowns are $H^\Lambda = \{H^I(\tau)\,,\, H_I(\tau)\}$

$\Rightarrow$ the eqm are translated to [MEESSEN,ORTÍN,PERZ,SHAHBAZI]

$$\begin{aligned}
\left[\partial_\Lambda\partial_\Sigma \log \mathsf{W} - 2\frac{H_\Lambda H_\Sigma}{\mathsf{W}^2}\right]\ddot{H}^\Sigma + \tfrac{1}{2}\partial_\Lambda\partial_\Sigma\partial_P \log \mathsf{W}\ \left[\dot{H}^\Sigma \dot{H}^P - \tfrac{1}{2}\mathcal{Q}^\Sigma\mathcal{Q}^P\right] & \\
-4\dot{H}_\Lambda\frac{\dot{H}^\Sigma H_\Sigma}{\mathsf{W}^2} + 8H_\Lambda\frac{(\dot{H}^P\tilde{H}_P)(\dot{H}^\Sigma H_\Sigma)}{\mathsf{W}^3} + 2\mathcal{Q}_\Lambda\frac{H^\Sigma\mathcal{Q}_\Sigma}{\mathsf{W}^2} & \\
-4\tilde{H}_\Lambda\frac{(H^\Sigma\dot{H}_\Sigma)^2}{\mathsf{W}^3} - 4\tilde{H}_\Lambda\frac{(H^\Sigma\mathcal{Q}_\Sigma)^2}{\mathsf{W}^3} &= 0
\end{aligned}$$

$$-\tfrac{1}{2}\partial_\Lambda\partial_\Sigma \log \mathsf{W}\left(\dot{H}^\Lambda\dot{H}^\Sigma - \tfrac{1}{2}\mathcal{Q}^\Lambda\mathcal{Q}^\Sigma\right) + \left(\frac{\dot{H}^\Lambda H_\Lambda}{\mathsf{W}}\right)^2 - \left(\frac{\mathcal{Q}^\Lambda H_\Lambda}{\mathsf{W}}\right)^2 = r_0^2$$

where $W = \mathrm{e}^{-2U}$, $\mathcal{Q}^\Lambda = \{p^I\,,\, q_I\}$, $H^\Sigma\mathcal{Q}_\Sigma = H^I\mathcal{Q}_I - H_I\mathcal{Q}^I$ and $\tilde{H}_\Sigma = \partial_\Sigma W$

General results

For all BHs $z^a(H)$ and $U(H)$ have the same expression

$\rightarrow$ what changes is the functional form of H^Λ:

- Supersymmetric (extremal) solutions:

$$H^\Lambda = \mathcal{Q}^\Lambda \tau + K^\Lambda$$

harmonic functions with poles in the charges

- Non-supersymmetric extremal solutions:

$$H^\Lambda = \tilde{\mathcal{Q}}^\Lambda \tau + K^\Lambda$$

harmonic functions with poles in $\tilde{\mathcal{Q}}^\Lambda = \tilde{\mathcal{Q}}^\Lambda(\mathcal{Q})$

- Non-supersymmetric non-extremal solutions:

$$H^\Lambda = K^\Lambda \cosh r_0\tau + \frac{\hat{\mathcal{Q}}^\Lambda}{r_0} \sinh r_0\tau$$

hyperbolic functions with $\hat{\mathcal{Q}}^\Lambda = \hat{\mathcal{Q}}^\Lambda(\mathcal{Q}, z_\infty, r_0)$

K^Λ is const. determined by imposing asymptotic flatness, absence of NUT-charge and definition of the scalars at infinity

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Outlook

All the solutions analyzed require $\dot{H}^I H_I - \dot{H}_I H^I = 0$

$\rightarrow$ This is a necessary condition **only** in the susy case

But for non-supersymmetric solutions

¿ what if this constraint is relaxed ?

New functional forms for the H^Λ would arise and new more complicated black hole solutions could appear

Further developments are expected but we could be close to a final resolution of the problem of finding BH solutions in supergravity