hvLf metrics in ungauged supergravity

Pablo Bueno Gómez

In collaboration with:

W. Chemissany, P. Meesen, T. Ortín & C. S. Shahbazi

arXiv:1209.4047 [hep-th]

Instituto de Física Teórica UAM/CSIC

Barcelona, October 19th, 2012

Contents

Gauge/gravity duality

Gauge theory $\stackrel{?}{=}$ Gravity/String theory

Same theory in different variables?

Paradigmatic example: **aDS/CFT correspondence**

IIB on $aDS_5 \times S^5 \iff \mathcal{N} = 4,\ D = 4$ Super-Yang-Mills

J.M. Maldacena Adv.Theor.Math.Phys. 2 (1998) 231-252

aDS/CFT correspondence

- **Conformal Field Theory**, invariant under: $x^{\mu} \to \lambda x^{\mu}$.
- Extra dimension u as an energy scale $\Rightarrow$ $u \to \dfrac{u}{\lambda}$

Most general $D+1$-dim metric with this symmetry and D-dim Poincaré invariance?

$$\Updownarrow$$

$$\boxed{ds^2 = \left(\frac{u}{L}\right)^2 \eta_{\mu\nu} dx^{\mu} dx^{\nu} + \frac{du^2}{u^2} L^2} \Longleftrightarrow \textbf{aDS}_{D+1} \textbf{ metric}$$

$$\Updownarrow$$

$$\boxed{ds^2 = \left(\frac{L}{r}\right)^2 \left(\eta_{\mu\nu} dx^{\mu} dx^{\nu} + dr^2\right)}$$

AdS/CFT generalizations

Less symmetric gauge theories ⟺ **Gravity/String theories in less symmetric spacetimes**

⇕

Recent application...

Modelling strongly coupled Condensed Matter systems!!

Hartnoll, Herzog, McGreevy, Charmousis, Gouteraux, Kim, Kiritsis, Meyer, Huijse, Sachdev, Swingle, Dong, Harrison, Kachru, Torroba, Wang, Shaghoulian...

Dynamical scaling behaviour

System characterized by a single length scale L(t), invariant under rescalings of spatial coordinates: $x^i \to L(t)x^i, \quad i \neq 0$.

$L(t) \sim t^{1/z}$

z $\Longleftrightarrow$ **universal dynamical exponent**, independent of most system details.

Fixed points governing phase transitions in some CM systems invariant under: $x^i \to \lambda x^i$, $t \to \lambda^z t$.

Hyperscaling violation

Henceforth, d is the number of spatial dimensions on which the dual theory lives, e.g. aDS$_4$ $\leftrightarrow$ $d = 2$

Hyperscaling $\Longleftrightarrow$ **All dimensionful quantities scale with their naive power of length.**

E.g. If $z \neq 1$, entropy density scales as: $S \sim T^{\frac{d}{z}}$

Certain systems present **hyperscaling violation**: $S \sim T^{\frac{(d-\theta)}{z}}$

$\Updownarrow$

Thermodynamic behaviour as if they lived in $d - \theta$ dimensions.

hvLf metrics

Hyperscaling-violating Lifshitz metrics (hvLf)

$$\boxed{ds^2_{d+2} = l^2 r^{-\frac{2(d-\theta)}{d}} \left[r^{-2(z-1)} dt^2 - dr^2 - dx^i dx^i \right]}, \quad i = 1, \dots, d$$

Most general spatially homogeneous and covariant under

$$x^i \to \lambda x^i, \quad t \to \lambda^z t, \quad r \to \lambda r, \quad ds^2_{d+2} \to \lambda^{2\theta/d} ds^2_{d+2}$$

They suffer from a null curvature singularity at $r \to \infty$ except for a specific set of parameter values. Tidal forces diverge as

$$C_{(\theta,z)} r^{2C_{(\theta,z)}+d}, \quad C_{(\theta,z)} = \frac{d(z-1)-\theta}{d-\theta}$$

hvLf metrics

hvLf metrics with $\theta \neq 0$ found so far as solutions to EMD type effective actions

$$S = \frac{1}{16\pi G_N} \int \sqrt{|g|} \{ R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - Z(\phi) F^{\mu\nu} F_{\mu\nu} - 2\Lambda - V(\phi) \}$$

Let's find some new hvLf metrics!

$D = 4$ SUGRA

The bosonic sector of any ungauged $D = 4$ SUGRA can be put in the form

$$I = \int d^4x\sqrt{|g|}\left\{R + \mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j + 2\Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}{}_{\mu\nu}F^{\Sigma\,\mu\nu} - 2\Re\mathrm{e}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}{}_{\mu\nu}\star F^{\Sigma\,\mu\nu}\right\}$$

$G_{ij}(\phi) \leftrightarrow$ Scalar metric: $+$definite metric on the scalar manifold

Number of scalar fields labeled by: $i, j, \ldots$

$\mathcal{N}_{\Lambda\Sigma} = \mathcal{N}_{\Lambda\Sigma}(\phi) \leftrightarrow$ *Period* matrix: complex, symmetric, -definite imaginary part

Number of vector fields labeled by: $\Lambda, \Sigma, \ldots$

$D = 4$ SUGRA

Static, spherically symmetric solutions

$$\Updownarrow$$

$$ds^2 = e^{2U(\tau)}dt^2 - e^{-2U(\tau)}\gamma_{\underline{mn}}dx^{\underline{m}}dx^{\underline{n}}$$

$$\gamma_{\underline{mn}}dx^{\underline{m}}dx^{\underline{n}} = \frac{d\tau^2}{W_{-1}(\tau)^4} + \frac{d\Omega^2_{-1}}{W_{-1}(\tau)^2}$$

$$d\Omega^2_{-1} = d\theta^2 + \sin^2\theta d\phi^2$$

Equations of motion $\Longrightarrow$ $W_{-1}(\tau) = \frac{\sinh(r_0\tau)}{r_0}$

$r_0 \leftrightarrow$ non-extremality parameter

BHs in $D = 4$ SUGRA

$$ds^2 = e^{2U(\tau)}dt^2 - e^{-2U(\tau)}\left[\frac{r_0^4}{\sinh^4(r_0\tau)}d\tau^2 + \frac{r_0^2}{\sinh^2(r_0\tau)}\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

Good number of solutions of this kind for different $\mathcal{N} = 2,\ D = 4$ SUGRA models known

$$\Updownarrow$$

Single, charged, static, spherically symmetric, asymptotically-flat, non-extremal BHs

- $\tau \in (-\infty, 0) \longleftrightarrow$ (outer (event) horizon, spatial infinity)
- $\tau \in (\tau_s, +\infty) \longleftrightarrow$ (singularity, inner (Cauchy) horizon)

Three times more solutions for free

E.O.M. admit a generalization of $\gamma_{\underline{mn}} dx^{\underline{m}} dx^{\underline{n}}$:

All the solutions $(U(\tau), \phi)$ obtained in the spherically symmetric case **are also solutions** if we set

$$\gamma^{\kappa}_{\underline{mn}} dx^{\underline{m}} dx^{\underline{n}} = \frac{d\tau^2}{W_\kappa^4} + \frac{d\Omega_\kappa^2}{W_\kappa^2}, \quad \kappa = (-1), 0, 1$$

$$d\Omega^2_{+1} = d\theta^2 + \sinh^2\theta d\phi^2 \;\Leftrightarrow\; W_{+1} = \frac{\cosh(r_0\tau)}{r_0}$$

$$\boxed{d\Omega_0^2 = d\theta^2 + d\phi^2 \;\Leftrightarrow\; W_0^{\pm} = \frac{e^{\mp r_0 \tau}}{r_0}}$$

Metric function

Behaviour of the metric function (*BH normalized*)

- $\lim_{\tau\to 0^-} e^{-2U} = 1$
- $e^{-2U} \overset{\tau\to\pm\infty}{\sim} \frac{S_\pm}{4\pi r_0^2} e^{\mp 2r_0\tau}$
- $e^{-2U}(\tau_s) = 0$ for some τ_s

$\kappa = 0$ solutions and hvLf behaviour

$$ds^2_{\pm} = e^{2U(\tau)}dt^2 - e^{-2U(\tau)}\left[e^{\pm 4r_0\tau} r_0^4 d\tau^2 + e^{\pm 2r_0\tau} r_0^2 \left(d\theta^2 + d\phi^2\right)\right], \quad (a = 1/r_0)$$

$$ds^2_{-} \overset{\tau\to+\infty}{\Longrightarrow} \mathcal{R}^2_i \times \mathbb{R}^2$$

$$\rho \equiv e^{-r_0\tau}, \;\; \tilde{t} \equiv \frac{4\pi r_0}{S_+} t, \quad x^1 \equiv \theta, \;\; x^2 \equiv \phi$$

$$\boxed{ds^2_{-} \overset{\tau\to-\infty}{\Longrightarrow} \frac{S_+}{4\pi}\rho^4\left[\rho^{-6} d\tilde{t}^2 - d\rho^2 - dx^i dx^i\right]}$$

$$\Updownarrow$$

hvLf with $z = 4, \;\; \theta = 6, \;\; l \sim r_0$

Schwarzschild

In this coordinates: $e^{-2U} = e^{2M\tau}$, with the event horizon (conventionally) at $\tau \to -\infty$

$$ds^2_- \Longleftrightarrow \mathcal{R}^2_i \times \mathbb{R}^2$$

$$e^{M\tau} \equiv \rho, \rho \in (0, +\infty) \quad \boxed{ds^2_+ = M^2\rho^4 \left\{\rho^{-6}dt^2 - d\rho^2 - d\theta^2 - d\phi^2\right\}}$$

$$\Updownarrow$$

hvLf with $z = 4$, $\theta = 6$, $l \sim r_0$ everywhere

Reissner-Nordström

Near-horizon behaviour fits in the general case (asymptotically $z=4$, $\theta=6$ for $\tau\to-\infty$ in ds^2_- and for $\tau\to+\infty$ in ds^2_+)

What about the **near-singularity** behaviour?

The $\kappa=0$ metric associated to the embedding of the RN BH in pure $\mathcal{N}=2$, $D=4$ SUGRA reads ($r\in(0,+\infty)$)

$$ds^2_{(\pm)}=\frac{(r-r_+)(r-r_-)}{r^2}dt^2-\frac{r_0^4r^2}{(r-r_\pm)(r-r_\mp)^5}dr^2-\frac{r_0^2r^2}{(r-r_\mp)^2}\left(d\theta^2+d\phi^2\right)$$

$$\boxed{ds^2_{(\pm)}\overset{\substack{r\to0\\(\tau\to\tau_s)}}{\Longrightarrow}\frac{r_+r_-}{r^2}dt^2-\frac{r_0^4r^2}{r_\pm r_\mp^5}dr^2-\frac{r_0^2r^2}{r_\mp^2}\left(d\theta^2+d\phi^2\right)}$$

$$\Updownarrow$$

hvLf with $z=3$, $\theta=4$

From limiting procedures

$d\Omega^2_{-1} \simeq d\Omega^2_0$ near $\theta = \pi/2$: $d\theta^2 + \sin^2\theta d\phi^2 \overset{\theta\to\pi/2}{\Longrightarrow} d\theta^2 + d\phi^2$

Near-singularity behaviour of **Reissner-Nordström** and **(negative mass) Schwarzschild BHs** around $\theta = \pi/2$

$$ds^2 \overset{\theta=\pi/2}{=} \frac{(r-r_+)(r-r_-)}{r^2} dt^2 - \frac{r^2}{(r-r_+)(r-r_-)} dr^2 - r^2\left(d\theta^2 + d\phi^2\right)$$

$$ds^2 \overset{\substack{r\to 0\\ \theta=\pi/2}}{\Rightarrow} \frac{r_+ r_-}{r^2} dt^2 - \frac{r^2}{r_+ r_-} dr^2 - r^2\left(d\theta^2 + d\phi^2\right) \Longleftrightarrow$$

hvLf $z = 3,\ \theta = 4$

$$ds^2 \overset{\theta=\pi/2}{=} \left(1 + \frac{2|M|}{r}\right) dt^2 - \left(1 + \frac{2|M|}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + d\phi^2\right)$$

$$ds^2 \overset{\substack{r\to 0\\ \theta=\pi/2}}{\Rightarrow} \frac{2|M|}{r} dt^2 - \frac{r}{2|M|} dr^2 - r^2\left(d\theta^2 + d\phi^2\right) \Longleftrightarrow$$

hvLf $z = 4,\ \theta = 6$

$$\frac{2r}{|M|} \equiv \rho^2, \quad \frac{4t}{|M|} \equiv \tilde{t}$$

Check out arXiv:1209.4047 [hep-th] if you want more (and if you don't, do it too!)

¡Moltes gràcies!

From smearing

Most general SUSY static solution of $\mathcal{N}=2$, $D=4$ SUGRA

$$ds^2 = e^{2U}dt^2 - e^{-2U}d\vec{y}^{\,2}_{(3)}; \quad e^{-2U} = \frac{1}{2}(H^0)^2 + 2(H_0)^2.$$

H_0, H^0 **real harmonic** functions in the flat transverse space such that $H^0\partial_m H_0 - H_0\partial_m H^0$, $m = 1,2,3$

Purely electric extremal RN BH $\Longleftrightarrow$ $H^0 = 0$, $H_0 = 1 + \frac{1}{\sqrt{2}}\frac{|q|}{|\vec{y}|}$

Harmonic functions depending on only one coordinate $y^3 \equiv \rho$ **?**
$\Longleftrightarrow$ **Smearing** of the spherically-symmetric solution in the (y^1, y^2) plane $\Longleftrightarrow$ *replace* $1/|\vec{y}|$ by ρ

$$ds^2 = \frac{1}{2}(H_0)^{-2}dt^2 - 2(H_0)^2\left[d\rho^2 + d\vec{y}^{\,2}_{(2)}\right]; \quad H_0 = 1 + \frac{1}{\sqrt{2}}|q|\rho$$

hvLf $z=3$, $\theta=4$ as $\rho\to\infty$, and everywhere if $H_0 = \frac{1}{\sqrt{2}}|q|\rho$

Give me more hvLf

Smearing of D0-D4 black holes embedded in the STU model

⇕

Near-horizon hvLf $z = 3,\ \theta = 4$; $z = 1,\ \theta = 2$; $z = 2,\ \theta = 3$; $z = 5/2,\ \theta = 7/2$ **...**

$\mathcal{N} = 2$ **D-dim SUGRA's single, charged, static, spherically symmetric, asymptotically-flat, non-extremal BHs**

⇕

Near-horizon hvLf $z = \frac{2d}{d-1},\ \theta = \frac{d(d+1)}{d-1}$