Supersymmetric BCS superconductivity

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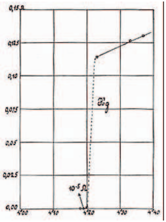
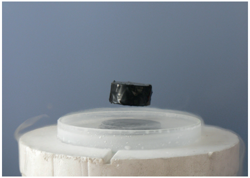
Outline

Review of superconductivity

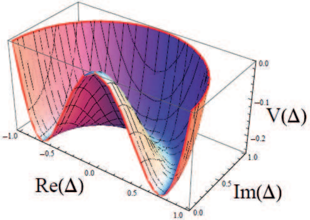




- London:

$$\vec{J} \propto \vec{A} \qquad \Rightarrow \qquad \begin{array}{l} E \propto \partial_t J \\ \nabla^2 B \propto B \end{array}$$

- Landau-Ginzburg: $\mathbf{U}(1)$ **spontaneous symmetry breaking** when $T < T_c$



$$\mathcal{F}_{LG} = \alpha(T - T_c)|\Delta|^2 + \frac{\beta}{2}|\Delta|^4 + \dots$$

- Bardeen-Cooper-Schrieffer: Cooper pairs

Motivation

- High T_c superconductors:

  The pairing mechanism is not well understood ⇐ it involves strong coupling.

  AdS/CFT is an new tool to study strongly coupled field theories.

Holographic Superconductor

Gravity	Superconductor
Black Hole	T
A_t	μ
"Hair"	Δ
$U(1)$ gauge	$U(1)$ global

This is typically supersymmetric.

Hartnoll, Herzog, Horowitz, Holographic superconductors

- Possible applications to real condensed matter systems with fermion and scalar quasiparticle excitations.
- SUSY softens divergences.

$$\mathcal{L} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi - m\psi^\dagger \psi + \underline{g^2(\psi\psi)(\psi^\dagger\psi^\dagger)}$$

- Add chemical potential for some conserved charge:
  "gauge field" $A_0 = \mu \Rightarrow$ Fermi Surface

Polchinski, Effective Field Theory and the Fermi Surface

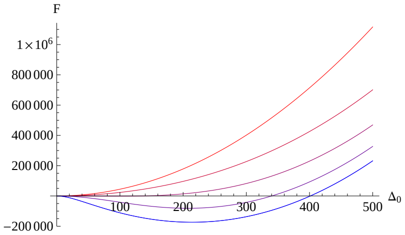
- Adding temperature: Euclideum with $\phi(\beta, x) = \phi(0, x)$, $\psi(\beta, x) = -\psi(0, x)$
- Perform a Hubbard-Stratonovich transformation: $\Delta = (\psi^\dagger\psi^\dagger)$

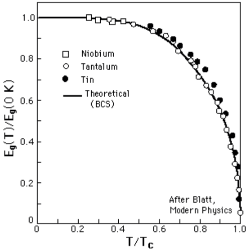
$$\mathcal{L} = \psi^\dagger \bar{\sigma}^0 (\partial^\tau + \mu)\psi + i\psi^\dagger \bar{\sigma}^i \partial^i \psi + m\psi^\dagger\psi - g^2\Delta(\psi\psi) - g^2\Delta^*(\psi^\dagger\psi^\dagger) + g^2|\Delta|^2$$

Classical potential V_{cl}

Energy eigenvalues: $\omega_\pm = \sqrt{(\omega_0 \pm \mu)^2 + |\Delta|^2} \qquad \omega_0 \equiv \sqrt{p^2 + m^2}$

$$\begin{aligned}\mathcal{F} = \quad & g^2\Delta^2 && \longrightarrow \text{Classical potential}\\ & + \int^{\Lambda_D} \frac{d^3p}{(2\pi)^3}\left(2\omega_0(p) - \omega_-(p) - \omega_+(p)\right) && \longrightarrow \text{Coleman-Weinberg}\\ & -\frac{2}{\beta}\int \frac{d^3p}{(2\pi)^3}\left(\log(1+e^{-\beta\omega_-(p)}) + \log(1+e^{-\beta\omega_+(p)})\right) && \longrightarrow \text{Thermal}\end{aligned}$$





SUSY BCS: μ for $U(1)_B$

$$K(\Phi,\Phi^\dagger) = \Phi_x^\dagger\Phi_x + \Phi_y^\dagger\Phi_y + g^2(\Phi_x^\dagger\Phi_x)^2 + g^2(\Phi_y^\dagger\Phi_y)^2 \qquad W = m\Phi_x\Phi_y$$

$$\Downarrow$$

$$\mathcal{L}_S = (1+4g^2|\phi_x|^2)\partial_\mu\phi_x^*\partial^\mu\phi_x - \frac{m^2|\phi_y|^2}{1+4g^2|\phi_x|^2} + (x\leftrightarrow y)$$

$$\mathcal{L}_F = i(1+4g^2|\phi_x|^2)(\psi_x^\dagger\bar\sigma^\mu\partial_\mu\psi_x) + 4ig^2(\psi_x^\dagger\bar\sigma^\mu\psi_x)\phi_x^*\partial_\mu\phi_x + \frac{g^2(\psi_x\psi_x)(\psi_x^\dagger\psi_x^\dagger)}{1+4g^2|\phi_x|^2}$$
$$+\left(\frac{2mg^2\phi_y\phi_x^*}{1+4g^2|\phi_x|^2}(\psi_x\psi_x) - \frac{1}{2}m\psi_x\psi_y + h.c.\right) + (x\leftrightarrow y)\ .$$

$U(1)_B$ baryonic symmetry: Φ_x and Φ_y have opposite charges.

$$\Delta_x = -\Delta_y \equiv \Delta \qquad \begin{aligned}\omega_F &= \sqrt{(\sqrt{p^2+m^2}\pm\mu)^2 + 4g^4\Delta^2}\\ \omega_S &= \sqrt{4g^4\Delta^2+m^2+p^2}\pm\mu\end{aligned}$$

SUSY BCS: μ for $U(1)_B$

$$\omega_F = \sqrt{(\sqrt{p^2+m^2} \pm \mu)^2 + 4g^4\Delta^2} \qquad \omega_S = \sqrt{4g^4\Delta^2 + m^2 + p^2} \pm \mu$$

Fermi surface defined by the minimum of ω_{F-}: $\sqrt{p_F^2 + m^2} = \mu \quad \Rightarrow \quad \mu > m$

$$\omega_S < 0 \qquad V_{\text{thermal}} = \frac{1}{\beta} \sum_{\omega} \int \frac{d^3p}{(2\pi)^3} \log(1 - e^{-\beta\omega_S}) \quad \text{is ill defined}$$

The occupation number of scalars with zero momentum goes to ∞ as $\mu \to m$

Bose-Einstein Condensation, which spoils BCS mechanism.

Try putting the theory on $S^1 \times S^3$:

- Scalars couple to curvature $\Rightarrow$ Extra mass term: $\left(m^2 + R^{-2}\right)(\phi_x^* \phi_x)$
- The scalar would be negligible if $1/R > \Lambda$.
- The integral over momentum is replaced by a discrete sum originating from the Kaluza-Klein modes of S^3.

$$\begin{array}{rcll} \text{Scalars:} & p^2 \longrightarrow & l(l+2)R^{-2} & \\ \text{Fermions:} & p^2 \longrightarrow & (l+1/2)^2 R^{-2} & \qquad l = 0, 1, 2\ldots \end{array}$$

R-symmetry

$$\begin{aligned} [Q_\alpha, R] &= Q_\alpha \\ [Q^\dagger_{\dot\alpha}, R] &= -Q^\dagger_{\dot\alpha} \end{aligned} \qquad \Longrightarrow \qquad R_\psi = R_\phi - 1$$

- $R_\phi = 0$ and $R_\psi = -1$ allows us to introduce μ for only fermions $\Rightarrow$ We avoid BEC but $R_W \neq 2 \Rightarrow m = 0$.
- R-symmetry must be non-anomalous to have a well defined μ.

$$\sum_{\text{fermions}} R^3 = 0$$

- Add more chiral fields e.g. Φ_{Z1}, Φ_{Z2}, with $R_{\phi_Z} = 2$ and canonical Kähler potential.
- The new scalars couple to $\mu \Rightarrow$ BEC
  Z sector is completely decoupled from X, Y sector.

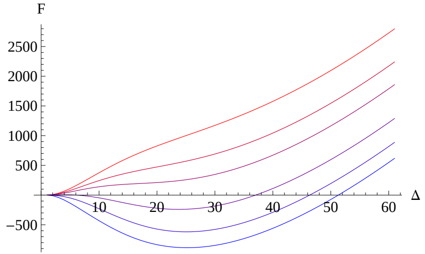
Energy eigenvalues:

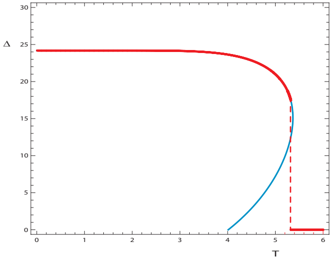
$$\omega^2_{S\ x,y} = p^2 + \frac{4g^4\Delta^2_{x,y}}{1+4g^2v^2_{x,y}}$$

$$\omega^2_{F\ x,y} = (p \pm \mu)^2 + \frac{4g^4\Delta^2_{x,y}}{\left(1+4g^2v^2_{x,y}\right)^2}$$

Classical Potential:

$$V_{cl} = g^2(1+4g^2v_x^2)|\Delta_x|^2 + (x \leftrightarrow y)$$





Conclusions

- We have been able to implement BCS theory in a SUSY field theory with $U(1)_R$ symmetry,
- Scalars make the phase transition first order rather than second order.
- SUSY softens divergences $V_{CW} \sim \Lambda_D^2 \to \log \Lambda_D$.

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Thank you for your attention